Analysis of the solution for the economic load dispatch by different mathematical methods and genetic algorithms: Case Study

Manoel Henrique Reis Nascimento¹, Jorge de Almeida Brito Junior², Carlos Alberto Oliveira de Freitas³, Nadime Mustafa Moraes⁴, Jandecy Cabral Leite⁵

¹Dr. Research Department, Institute of Technology and Educaction Galileo of the Amazon (ITEGAM)
²Dr. Research Department, Institute of Technology and Educaction Galileo of the Amazon (ITEGAM)
³M.Sc. Research Department, Institute of Technology and Galileo Educaction da Amazònia (ITEGAM)
⁴Dr. Research Department, University of the State of Amazonas (UEA)
⁵Dr. Research Department, Institute of Technology and Galileo Educaction da Amazônia (ITEGAM)

Email: hreys@itegam.org.br, jorge.brito@itegam.com.br, carlos.freitas@itegam.com.br, nmoraes@uea.edu.br, jandecy.cabral@itegam.org.br

ABSTRACT

The optimization of Economic Load Dispatch (ELD) is one of the most important tasks in Power Plants (PP). The paper objective is to analyze a new application of the computational optimization technique by Genetic Algorithms including Turning Off the motors with greatest losses. The incremental cost of fuel is used to determine the best parameters of active power of each ith generator unit, ensuring that the demand and total losses are equal to the total generated power but minimizing the total cost of fuel. Materials and methods have been developed to solve the economic load dispatch, among them: lambda iteration, gradient method, Newton and so others. The results found for this case study, with the new application of Optimization by Genetic Algorithms were outstanding having a reduction of 19.88% in the total fuel cost, comparing to classical methods that distribute the generation of energy among all motors, including the least efficient ones. This method helps the expert in the decision making of preventive maintenance of machines that are not being used in the moment of multi-objective optimization, improving not only the efficiency of motors but also of the power plant generation planning.

Keywords: Genetic Algorithms, Load Economic Dispatch, Mathematical methods, Power plants.

I INTRODUCTION

The problem of economic load dispatch (ELD) is to minimize the total cost of generation and at the same time meeting the demand for electricity from the plant. The classic economic dispatch problem is to provide the required amount of power at the lowest cost, to meet the demand and the operational restrictions. This is a very complex problem to solve because of its large size, a nonlinear objective function, and a large number of restrictions. Several techniques such as Integer Programming [1], the Dynamics Programming [2] and Lagrange Functions [3] have been used to solve the problem of economic load dispatch. Other optimization methods such as Simulated Annealing [4], Neural Networks [5], Genetic Algorithms [6] Particle Swarm Optimization [7] and Tabu Search Algorithm [8], are also practiced to solve the ELD problem. There have also been developed methods based on mathematical approaches to offer a quicker solution [9]. Multiobjective evolutionary algorithms have also been applied to the problem at hand [10]. There have conducted research to minimize costs [11] including emission restrictions form solving the economic dispatch and the selection of machines. Recently, there have been successfully employed numerical methods that are more convenient and novel techniques for solving the ELD optimization problems [12]. In [13] it is presented an Particle Swarm Optimization (PSO) method with an aged leader and adversaries (ALC-PSO) to solve the optimization problem of reactive power dispatch. According to [14], due to the non-convexity of the problem of ELD, it becomes difficult to ensure the global optimum. In [15] an evolutionary algorithm known as "Cuckoo Search" was applied to problems of economic load dispatch when non-convexity occurs. In [16] a methodology to solve the ELD problem is presented, considering the uncertainty of reliability of of wind turbines generation. The proposed method is compared with the Monte Carlo Simulation (MCS) method.
In [17] it is stated that linear programming is the strategy that has the lowest increase in generation costs, but the strategy based on genetic algorithms is the best for minimizing operating costs and the total energy demand of the system.

According to [18], the dynamic economic dispatch is one of the most complicated nonlinear problems to solve in power systems due to its non convexity, mainly due to the effect of "valve point" in the cost functions of the generating units, limit gradient and transmission losses. Therefore, the proposal of a method for an effective solution to this optimization problem is of great interest.

The objective of the article is to use a new application of the computational optimization technique by Genetic Algorithms (GA). Here we combine the incremental fuel cost methods to determine the best active power parameters of each generating unit to solve the ELD problem. How to analyze the application of the computational optimization technique by GA, including the deactivation of engines with higher losses. The incremental cost of fuel is used to determine the best active power parameters of each generating unit, ensuring that total demand and losses are equal to the total power generated, but minimizing the total cost of fuel.

In the current literature during the process of optimization is used all the capacity of PP, distributing the demand for each \( i_{th} \) plant generation unit, including the least efficient ones. The contribution of this article is the implementation of GA in order to satisfy the demand of active power with changes to turn off the least efficient machines that will not be necessary in the moment of multi-objective optimization. This method helps the expert in the decision making of preventive maintenance of machines that are not being used in the moment of multi-objective optimization, improving not only the efficiency of motors but also of the PP generation planning.

II MATERIALS AND METHODS

II.1 ECONOMIC LOAD DISPATCH

As was pointed above the classic economic dispatch problem it is to supply the required amount of power at the lowest possible cost. The problem of minimize the cost of fuel in power plants can be raised mathematically as follows: [15]:

\[
F = \text{Min}_{P_i} \sum_{i=1}^{n} (a_i + b_i P_i + c_i P_i^2)
\]  

(1)

The above expression is dependent on the balance of equality restrictions of actual power.

II.2 ECONOMIC LOAD DISPATCH TAKING INTO ACCOUNT THE "VALVE POINT" EFFECT

The cost function of the fossil fuel consumed in a generating unit is obtained from data points taken during testing of "performance" of the unit, when the input and output data are measured as the unit changes its operation region.

In the case of steam turbines these effects occur each time the intake valve in a steam turbine begins to open, and produces a ripple effect on the power curve versus fuel consumption of the unit. Generation units based on multivalve steam turbines are characterized by a complex nonlinear function of fuel cost. This is mainly due to undulations induced by loading because of the throttle valve or "valve point". To simulate this complex phenomenon, a sinusoidal component is superimposed on the quadratic curve of the engines. [19], as shown in Figure 1.

![Figure 1: Cost function taking into account the “valve point” effect with 5 valves.](image)

Source: [20].

The mathematical expression of cost considering the "valve point" effect is as follows: [21]:

\[
f_i(P_{iD}) = a_i P_{iD}^2 + b_i P_{iD} + c_i + |d_i \sin(f_i (P_{iD}^{min} - P_{iD}))| \left( \frac{\$/h}{h} \right)
\]  

(2)

Where \( a_i, b_i \) and \( c_i \) are the fuel cost coefficients of the \( i_{th} \) generating unit, and \( d_i \) and \( f_i \) are the fuel cost coefficients of the \( i_{th} \) generating unit due to the "valve point" effect.

Basu [22] states that the function of fuel costs of each generating unit, taking into account the effects of "valve point" is expressed as the sum of a quadratic function and a sine function. The total fuel cost in terms of real power can then be expressed as:

\[
F = \sum_{i=1}^{n} \sum_{m=1}^{m} d_i m |a_i + b_i P_m + c_i P_m^2 + \sin(e_i (P_{iD}^{min} - P_{iD}))| \left( \frac{\$/h}{h} \right)
\]  

(3)

Where \( d_i \) and \( e_i \) are the fuel cost coefficients of the \( i_{th} \) generating unit due to the effect of "valve point".

II.3 ECONOMICAL LOAD DISPATCH CONSTRAINTS

In this paper, several restrictions are considered:

- Equal power balance constrains.

For stable operation, the real power of each generator is limited by the lower and upper limits. The following equation is the equality of balance of power restriction [23]:

\[
\sum_{i=1}^{n} P_i - P^D - P^L = 0
\]  

(4)

where \( P_i \) is the output power of each generator \( i \), \( P^D \) is the load demand and \( P^L \) are transmission losses.

In other words, the total power generation has to meet the total demand \( P^D \) and the actual power losses in transmission lines \( P^L \), ie:

\[
\sum_{i=1}^{n} P_i = P^D + P^L
\]  

(5)
The calculation of power losses $P_L$ involves the solution of the load flow problem, which has equality constraints in the active and reactive power on each bar as follows [24]:

$$P_L = \sum_{i=1}^{n} B_i P_i^2$$  \hspace{1cm} (6)

To model the transmission losses, a simplification is applied, setting them as a function of the generators output through Kron’s loss coefficients derivatives of the Kron formula for losses.

$$P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} B_{ij} P_i P_j + \sum_{i=1}^{M} B_{0i} P_i + B_{00}$$  \hspace{1cm} (7)

Where $B_{ij}$, $B_{0i}$ and $B_{00}$ are the energy loss coefficients in the transmission network. It can be obtained a reasonable accuracy when the actual operating conditions are close to the base case, where the B coefficients were obtained [24].

- An inequality constraint in terms of generating capacity. For stable operation, the actual power of each generator is limited by upper and lower limits. The constrain in the limits of inequality in the generator output is:

$$P_{\min,i} \leq P_i \leq P_{\max,i}$$  \hspace{1cm} (8)

Where:
- $P_i$ – Output power of the $i$ generator
- $P_{\min,i}$ – Minimal Power of the $i$ generator
- $P_{\max,i}$ – Maximal Power of the $i$ generator

- An inequality constraint in terms of fuel delivery. At each interval, the amount of fuel supplied to all units must be less than or equal to the fuel supplied by the seller, i.e., the fuel delivered to each unit in each interval should be within its lower limit $F_{\min}$ and its upper limit $F_{\max}$, so that [22]:

$$F_{\min,m} \leq F_i \leq F_{\max,m}, \; i \in N, m \in M$$  \hspace{1cm} (9)

Where
- $F_i$ – Fuel supplied to the engine $i$ at the interval $m$
- $F_{\min,i}$ – Minimal limit of fuel supplied to $i$ machine
- $F_{\max,i}$ – Maximal limit of fuel supplied to $i$ machine
- $F_{D,m}$ – Total Fuel supplied at the interval $m$

- An inequality constraint in terms of fuel storage limits. The fuel storage limit of each unit in each interval should be within its lower limit $V_{\min}$ and the upper limit $V_{\max}$, so that [22]:

$$V_{\min,i} \leq V_{im} \leq V_{\max,i}$$  \hspace{1cm} (10)

$$V_{im} = V_{(m-1)} + F_{im} F_{im} - \sum_{m=1}^{M} [\eta_i + \delta_i P_i + \mu_i P_i^2 + |\lambda_i| \sin|\rho_i (P_{\min,i} - P_i)|]$$  \hspace{1cm} (11)

Where $\eta_i$, $\delta_i$ and $\mu_i$ are the coefficients of fuel consumption of each generating unit and $\lambda_i$ and $\rho_i$ are the coefficients of consumption of each generating unit taking into account the effect of strangulation or "valve point".

### II.4 THE ECONOMIC LOAD DISPATCH OPTIMIZATION PROBLEM

There are different mathematical and computational methods to solve the ELD problem that will be briefly described here.

#### II.4.1 FUEL INCREMENTAL COST METHOD

The incremental fuel cost can be obtained from the following equation [25]:

$$IC_i = (2. a_i P_{gi} + b_i) \$/hr$$  \hspace{1cm} (12)

Where $IC_i$ is the incremental fuel cost $a_i$ are the values of the different points of the actual curve of the incremental cost and $b_i$ are the values of the points on the approximated curve (linear) of incremental cost. $P_{gi}$ is the total power generation [26].

The curve of incremental fuel cost is shown in the following figure:

![Incremental Cost Curve of an electricity generator. Source: [27].](image)

For purposes of load dispatching the cost is generally approximated to one or more quadratic segments, then the fuel cost curve in active power generation, takes a quadratic form

#### II.4.2 ITERATION LAMBDA METHOD

One of the most popular traditional techniques to solve the problem of economic load dispatch (ELD), minimizing the cost of generating unit is the lambda iteration method. Although the computational procedure of the lambda iteration technique is complex, it converges rapidly for this type of optimization problem [28].

The lambda iteration method is more conventional to deal with minimizing the cost of power generation to any demand. For a large number of units, lambda iteration method is more accurate and more accurate incremental cost curves of all generating units are obtained.

Below is detailed the lambda iteration algorithm for economic load dispatch.
The steps to resolve the algorithm of the lambda (λ) iteration method are:

1. To read the problem data:
   * The cost coefficients (a, b, c)
   * The Power Limits
   * The Power Demand
2. Assume an initial value of λ and Δλ for using the equations of costs
3. Calculate the power generated by each unit Pgi
4. Check the limits of generation of each unit:
   - if \( P_{gi} > P_{gi}^{max} \), set \( P_{gi} = P_{gi}^{max} \)
   - if \( P_{gi} < P_{gi}^{min} \), set \( P_{gi} = P_{gi}^{min} \)
5. Calculate the power generated.
6. Calculate the difference of powers given by the following equation:
   \[
   \Delta P = \sum_{i=0}^{Ng} P_{gi} - P_d
   \] (13)
7. if \( \Delta P < \varepsilon \) (tolerance value), then stop calculations and estimate the cost of generation. Otherwise, go to the next stage.
8. if \( \Delta P > 0 \), then \( \lambda = \lambda - \Delta \lambda \)
9. if \( \Delta P < 0 \), then \( \lambda = \lambda + \Delta \lambda \)
10. Repeat the procedure from stage 3

II.4.3 SEQUENTIAL QUADRATIC PROGRAMING

An efficient and accurate solution to the problem of economic dispatch does not depend only on the size of the problem, in terms of the number of constraints and variables of the project; it also depends on the characteristics of the objective function and constraints.

When both objective functions and constraints are linear functions of the variables of the project, the ELD is known as a linear programming problem.

The quadratic programming problem (QPP) refers to the minimization or maximization of a quadratic objective function that is constrained linearly. The most difficult problem is the problem of nonlinear programming in which the objective function and constraints can be nonlinear functions of variables of the problem.

The solution to the latter problem requires an iterative process to obtain a search direction at each iteration procedure. This solution can be found by solving a quadratic programming subproblem. The methods for solving these problems are commonly referred to as Sequential Quadratic Programming (SQP) which is a nonlinear optimization method, where a QP subproblem is solved by iterations; They are also known as iterative quadratic programming, recursive quadratic programming or restricted variable metric method.

The SQP is in many cases superior to other methods for nonlinear constrained optimization programming, possessing advantages in terms of efficiency, accuracy and success in obtaining solutions to many problems available in the literature[30].

II.4.4 QUADRATIC PROGRAMMING ALGORITHM

It can be applied to optimization problems with quadratic objective functions and nonlinear constraints. In many problems the goal is quadratic and constraints are also quadratic, so they must become linear [31]. Nonlinear equations and inequalities are solved through the following steps:

Step 1: To initialize the procedure it is necessary to set the lower limit of generation of each plant as well as evaluating the incremental transmission losses coefficients and update the demand.

\[
P_i = P_i^{min}, \quad x_i = 1 - \sum_{j=1}^{n} B_{ij} P_i
\] (14)

and

\[
P_D^{new} = P_D + P^{old}_L
\] (15)

Step 2: Replace the coefficients of incremental costs and solve the set of linear equations to determine the incremental fuel cost \( \lambda \) as:

\[
\lambda = \frac{\sum_{i=1}^{n} 0.5 \times \frac{P_i^{new}}{x_i}}{\sum_{i=1}^{n} 0.5 \times \frac{x_i}{P_i}}
\] (16)

Step 3: Determine the power of each machine.

\[
p_i^{new} = \frac{\lambda \times x_i}{2 \times \frac{P_i^{new}}{x_i}}
\] (17)
If the machine violates its limits, that limit should be fixed and only the remaining plants should be considered in the next iteration.

Step 4: Check convergence

\[ |\sum_i^n (P_i - PD^{\text{new}} - P_i) | \leq \epsilon \]  

(18)

\( \epsilon \) - is the value of the tolerance in the power balance.

Step 5: Run steps from 2 to 4 until convergence is reached. For all the above four steps the objective function is quadratic, but also the constraints are quadratic, and that restrictions should be converted to linear:

Minimize: \( XH X^T + f^T X \)  

(19)

Subjected to:

\[ KX \leq R \quad , \quad X^{\text{min}} \leq X \leq X^{\text{max}} \]

\[ X = [x_1, x_2, x_3, \ldots \ldots , x_{n}]^n \]

\[ f = [f_1, f_2, f_3, \ldots \ldots , f_{n}]^n \]

\[ R = [R_1, R_2, R_3, \ldots \ldots , R_{n}]^T \]

\( H \) is the Hessian matrix of size \( n \times n \) and \( A \) is the \( m \times n \) matrix, representing inequalities. For ELD considering losses the quadratic programming algorithm can be implemented effectively, defining the matrices \( H, f, K \) and \( R \):

\[ H = \text{diag} \left( \frac{a_2}{x_1}, \frac{a_2}{x_2}, \ldots \ldots , \frac{a_n}{x_n} \right) \]  

(20)

\[ f = \begin{bmatrix} b_1 \frac{b_2}{x_1} \frac{b_3}{x_2} \ldots \ldots \frac{b_n}{x_n} \end{bmatrix} \]

\( K \) is a matrix: \( 1 \times n \) \( K = [1, 1, \ldots \ldots , 1] \), \( y = R = PD + P_L^{\text{old}} \)

II.4.5 NEWTON'S METHOD

The economic dispatch can also be solved by the observation that the aim is always to generate that is getting \( \nabla L_x = 0 \).

Because this is a vector function, the problem may be formulated as a correction attempt that conducts exactly the gradient to zero (ie, a vector whose elements are zero). Newton's method can be used to accomplish this. This method for a function of several variables is developed as follows [32]:

Assume that a \( g(x) \) function will be converted to zero. The function \( g \) is a vector and the unknowns, \( x \), are also vectors. Then to use Newton's method, it is necessary to do the following:

\[ g(x + \Delta x) = g(x) + [g'(x)] \Delta x = 0 \]  

(21)

If the function is defined as:

\[ g(x) = \begin{bmatrix} g_1(x_1, x_2, x_3) \\ g_2(x_1, x_2, x_3) \\ g_3(x_1, x_2, x_3) \end{bmatrix} \]  

(22)

Then:

\[ g'(x) = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \frac{\partial g_1}{\partial x_3} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & \frac{\partial g_2}{\partial x_3} \\ \frac{\partial g_3}{\partial x_1} & \frac{\partial g_3}{\partial x_2} & \frac{\partial g_3}{\partial x_3} \end{bmatrix} \]  

(23)

That is the well-known Jacobian matrix. The adjustment of each step is then:

\[ \Delta x = -\left[ g'(x) \right]^{-1} g(x) \]  

(24)

Now, if the function \( g \) is the gradient vector \( \nabla L_x \) then:

\[ \Delta x = -\nabla \left( \frac{\partial}{\partial x} \nabla L_x \right) \]  

(25)

For the ELD problem, the expression to be used is:

\[ L = \sum_i^n F_i(P_i) + \lambda (P_{\text{load}} - \sum_i^n P_i) \]  

(26)

\( \nabla L \) has already been defined. The Jacobian matrix now becomes into a compound of second derivatives and it is called the Hessian matrix:

\[ \frac{\partial}{\partial x} \nabla L_x = \begin{bmatrix} \frac{d^2 L}{dx_1^2} & \frac{d^2 L}{dx_1 dx_2} & \ldots \ldots & \frac{d^2 L}{dx_1 dx_n} \\ \frac{d^2 L}{dx_2 dx_1} & \frac{d^2 L}{dx_2^2} & \ldots \ldots & \frac{d^2 L}{dx_2 dx_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{d^2 L}{dx_n dx_1} & \frac{d^2 L}{dx_n dx_2} & \ldots \ldots & \frac{d^2 L}{dx_n^2} \end{bmatrix} \]  

(27)

Generally, the Newton’s method will offer a setting that is much closer to the minimum generation cost than that offered by the gradient method

II.4.6 DYNAMIC PROGRAMMING METHOD

The application of digital methods for solving a variety of problems of control and dynamic optimization in the late 50s led to Richard Bellman and his collaborators to develop the dynamic programming.

These techniques are useful in solving a variety of problems and can greatly reduce the computational effort to find the best paths or control policies.

The theoretical mathematical background, based on the calculus of variations, is quite difficult. However the applications are not, since they depend on the will to express the particular problem of optimization in appropriate terms to a formulation of dynamic programming (DP) [33].

In the field of programming power generation systems, DP techniques for economic dispatch of power systems have been developed. If it considered the "valve point" effect in the engine curve, then it is necessary to work with not convex functions if high precision is required. In this case is not recommended to use the incremental cost method, because there are several values of output power for any given value of incremental cost.

Under such circumstances, there is a way to find an optimal dispatch using dynamic programming (DP). The solution of dynamic programming for the economic dispatch has to be considered as a location problem.

With this approach, not only a single set of optimal power output of the generator is calculated for a specific total load set, but a set of outputs at discrete points to a range of load values is also generated. A common problem to ELD with dynamic programming is the poor performance of control of generators.

The only way to produce a load dispatch acceptable to the control system as well as be the best economically is to add the limits of ramp rate into the economic dispatch.
This requires a short interval load forecasting to determine the best load requirements and ramp loading units with major probability. This problem can be setted as follows [34]:

Given a load to be generated to time intervals from \( t = 1 \) \( \ldots \) \( t_{\text{max}} \) with load levels of \( P_{\text{load}}^t \) and \( N \) generators on-line for supply the power:

\[
\sum_{t=1}^{N} P_i^t = P_{\text{load}}^t
\]

Each unit must obey a limit ratio, such that:

\[
P_i^{t+1} = P_i^t + \Delta P_i
\]

and

\[
-\Delta P_i^{\text{max}} \leq \Delta P_i \leq \Delta P_i^{\text{max}}
\]

Then, units must be programmed to minimize the cost of power delivery during the period in which:

\[
f_{\text{total}} = \sum_{t=1}^{t_{\text{max}}} \sum_{i=1}^{N} F_i(P_i^t)
\]

Restricted to:

\[
\sum_{t=1}^{N} P_i^t = P_{\text{load}}^t
\]

for \( t = 1 \ldots t_{\text{max}} \) and:

\[
P_i^{t+1} = P_i^t + \Delta P_i
\]

with:

\[
-\Delta P_i^{\text{max}} \leq \Delta P_i \leq \Delta P_i^{\text{max}}
\]

III ALGORITHMS GENETIC (GA)

GA algorithms borrow the analogous biological terms for each step. GAS maintain a population of parameter set solutions and iterate on the complete population. Each iteration is called a generation [35].

The problem a parameter set, including its environment, inputs, and outputs, is represented by a fixed length string of symbols, usually from the binary alphabet \((0, 1)\). The string, called a chromosome, represents a single solution point in the problem space. The chromosome string consists of genetic material in specific locations, called loci. Each location contains a symbol or series of symbols, called genes, which assume values, called alleles [36].

Evaluation of the chromosome string is accomplished by decoding the encoded symbols and calculating the objective function for the problem using the decoded parameter set. The result of the objective function calculation is used to calculate the value of the string with respect to all other chromosome strings within the population. This raw value measure is called the string’s fitness value and can be calculated in any number of ways based on the goal of the optimization. For maximization problems, the fitness value could assume the value of the objective function or payoff cost itself. For minimization problems, the fitness value could assume the value of the lowest value of the associated objective function. The fitness function is used to provide a measure of how individuals have performed in the problem domain. In the case of a minimization problem, the best fit individuals will have the lowest value of the associated objective function. The fitness function is normally used to transform the objective function value into a measure of relative fitness. The fitness function is defined as [35].

\[
\text{Fit}(x) = g(f(x))
\]

where

\[f(x)\] is the objective function

\[g(Q)\] transforms the value of \[f(x)\] to non-negative number.

Based on fitness criterion, poorer individuals are gradually taken out, and better individuals have a greater possibility of
conveying genetic information the next generation. Also, the best or elite member of a new generation is determined using the fitness criterion. This procedure is called elitism. The elitist procedure guarantees that the best solution so far obtained in the search is retained and used in the following generation, and thereby ensuring no good solution already found can be lost in the search process [35].

IV SOLUTION TO ELD ACCORDING TO INCREMENTAL COST OF FUEL AND LAMBDA ITERATION (λ) METHODS AND GENETIC ALGORITHMS, CASE STUDY

The problem to be solved by genetic algorithms can be formulated as follows:

- Minimize the equation (1) of the fuel cost of the \( i \)th engine;
- Considering the power losses are calculated by the expression (7);
- And the constraints used in the equation (8).

The power plant selected for the case study consists of 10 engines. The engine characteristics are shown in Table 1.

For determining the coefficients \( a, b \) and \( c \) there was conducted and experiment design running the engines to different powers and measuring the fuel consumption. Then were plotted the curves of power versus fuel cost and by a regression method the coefficients \( a, b \) and \( c \) were obtained

Table 1: Characteristic data of the motors from the case study power plant.

<table>
<thead>
<tr>
<th>Motor</th>
<th>( a_i ) ($/MWh)</th>
<th>( b_i ) ($/MWh)</th>
<th>( c_i ) ($/h)</th>
<th>( P_{\text{min}} ) (MW)</th>
<th>( P_{\text{max}} ) (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0077</td>
<td>7.00</td>
<td>240.00</td>
<td>0.66</td>
<td>3.35</td>
</tr>
<tr>
<td>2</td>
<td>0.0095</td>
<td>10.00</td>
<td>200.00</td>
<td>0.90</td>
<td>3.70</td>
</tr>
<tr>
<td>3</td>
<td>0.009</td>
<td>8.50</td>
<td>220.00</td>
<td>0.80</td>
<td>3.60</td>
</tr>
<tr>
<td>4</td>
<td>0.009</td>
<td>11.00</td>
<td>200.00</td>
<td>0.66</td>
<td>3.35</td>
</tr>
<tr>
<td>5</td>
<td>0.0088</td>
<td>10.50</td>
<td>220.00</td>
<td>0.72</td>
<td>3.45</td>
</tr>
<tr>
<td>6</td>
<td>0.0075</td>
<td>12.00</td>
<td>120.00</td>
<td>0.66</td>
<td>2.97</td>
</tr>
<tr>
<td>7</td>
<td>0.0075</td>
<td>14.00</td>
<td>130.00</td>
<td>0.88</td>
<td>3.50</td>
</tr>
<tr>
<td>8</td>
<td>0.0075</td>
<td>14.00</td>
<td>130.00</td>
<td>0.754</td>
<td>3.33</td>
</tr>
<tr>
<td>9</td>
<td>0.0075</td>
<td>14.00</td>
<td>130.00</td>
<td>0.90</td>
<td>3.90</td>
</tr>
<tr>
<td>10</td>
<td>0.0075</td>
<td>14.00</td>
<td>130.00</td>
<td>0.56</td>
<td>2.35</td>
</tr>
</tbody>
</table>


Loss coefficients \( (B_m) \) are given by a square matrix of size \( n \times n \), where \( n \) is the number of engines.

Table 2: Matrix of losses of the 10 engines from the power plant (all values must be multiplied by 1e-4).

\[
B_m = \begin{bmatrix}
 0.14 & 0.17 & 0.15 & 0.19 & 0.26 & 0.22 & 0.34 & 0.38 & 0.43 & 0.45 \\
 0.17 & 0.60 & 0.13 & 0.16 & 0.15 & 0.20 & 0.23 & 0.56 & 0.25 & 0.51 \\
 0.15 & 0.13 & 0.65 & 0.17 & 0.24 & 0.19 & 0.25 & 0.38 & 0.43 & 0.45 \\
 0.19 & 0.16 & 0.17 & 0.71 & 0.30 & 0.25 & 0.43 & 0.56 & 0.23 & 0.51 \\
 0.26 & 0.15 & 0.24 & 0.30 & 0.69 & 0.32 & 0.18 & 0.37 & 0.42 & 0.48 \\
 0.22 & 0.20 & 0.19 & 0.25 & 0.32 & 0.38 & 0.97 & 0.55 & 0.27 & 0.58 \\
 0.22 & 0.20 & 0.19 & 0.25 & 0.32 & 0.85 & 0.97 & 0.55 & 0.27 & 0.58 \\
 0.19 & 0.70 & 0.13 & 0.18 & 0.16 & 0.21 & 0.28 & 0.56 & 0.23 & 0.51 \\
 0.26 & 0.15 & 0.24 & 0.30 & 0.69 & 0.32 & 0.18 & 0.37 & 0.42 & 0.48 \\
 0.15 & 0.13 & 0.65 & 0.17 & 0.24 & 0.19 & 0.25 & 0.38 & 0.43 & 0.45 \\
\end{bmatrix}
\]

Source: [20]

The solution report offers the input parameters to run the program, such as power demand, minimum and maximum power of the machines and the results of the total cost of fuel, total loss of power and optimum power for each one of the machines in the plant to meet the load demand. In Figures 4-6 the most important graphics generated by genetic algorithm in MATLAB are offered.

V ANALYSIS AND DISCUSSION OF RESULTS

This restriction is related to the maximum capacity of the set of all machines, ensuring that the capacity demanded is less than the maximum generating capacity of the plant.

A population of 300 individuals and 1500 generations was used.

The results after running the program are:

<table>
<thead>
<tr>
<th>ELD using GA</th>
<th>Classical GA</th>
<th>Modified GA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Demand: 20 MW</td>
<td>20 MW</td>
<td>20 MW</td>
</tr>
<tr>
<td>Minimal Power: 0.56 MW</td>
<td>0.56 MW</td>
<td>0.56 MW</td>
</tr>
<tr>
<td>Maximal Power: 3.9 MW</td>
<td>3.7 MW</td>
<td>3.7 MW</td>
</tr>
<tr>
<td>Fuel Cost: R$ 1925.85</td>
<td>R$ 1540.83</td>
<td>R$ 1540.83</td>
</tr>
<tr>
<td>Power losses: 0.01 MW</td>
<td>0.01 MW</td>
<td>0.01 MW</td>
</tr>
</tbody>
</table>

Power of each Motor:

\[
\begin{align*}
P_1 &= 3.33 \\
P_2 &= 3.70 \\
P_3 &= 3.60 \\
P_4 &= 0.71 \\
P_5 &= 3.45 \\
P_6 &= 3.44 \\
P_7 &= 1.11 \quad \text{turning of} \\
P_8 &= 0.87 \\
P_9 &= 0.92 \quad \text{turning of} \\
P_{10} &= 0.88 \\
\end{align*}
\]

Total Power: 20.01


The solution report offers the input parameters to run the program, such as power demand, minimum and maximum power of the machines and the results of the total cost of fuel, total loss of power and optimum power for each one of the machines in the plant to meet the load demand. In Figures 4-6 the most important graphics generated by genetic algorithm in MATLAB are offered.

Figure 4: Values of the fitness function versus number of generations.

Source: Authors, (2018).

Figure 5: Best individual of each one of the variables

Source: Authors, (2018).

Figure 6: Number of offspring for each individual during the selection of the results.

Source: Authors, (2018).
The inequality of the powers of the selected engines has much to do with the loss coefficients of each motor, since the algorithm selects the engines that have fewer losses. In figures 7 and 8 are shown the graphics of power generator use and the power generation cost, outstanding the turned off of the motors that are not used for cover the power demand.

VI CONCLUSIONS

This paper presented an analysis of the problem of economic load dispatch ELD and different mathematical methods for solving the problem. Conventional methods such as lambda iteration method converge rapidly, but the complexity increases with increasing system size.

In addition, lambda iteration method always required to provide or meet the power output of a generator and then take an incremental cost for this generator. In cases where the cost function is much more complex, can be used the Newton's method.

If the input-output curves are not convex, then can be used dynamic programming to solve economic dispatch. Therefore, different methods have different applications. In this article the problem of economic load dispatch was resolved by the incremental cost of fuel and the lambda iteration methods. The lambda iteration method using genetic algorithms is a simple way to solve the problem of economic load dispatch successfully. In this paper were introduced some modifications to the genetic algorithm, such a way that can be turned off the motors with less efficiency.

VII ACKNOWLEDGEMENTS

To the Institute of Technology and Education Galileo of the Amazon (ITEGAM), to the Fundação de Amparo à Pesquisa State of Amazonas (FAPEAM) and to the Federal University of Pará (UFPA) for the financial and technical support to this research.

VIII REFERENCES


