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STATE FUNCTION Φ(U,E,t): CONNECTION OF ADVECTION, DISPERSION AND TURBULENCE IN NATURAL STREAMS AT DYNAMIC EQUILIBRIUM

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ABSTRACT

The Navier-Stokes equation has been a standard of calculus in hydrodynamics; however, its conceptual and computational problems make it necessary to look for better alternatives. One of the causes of this failure is that it is a "reductionist" model, which does not involve the emergence of new laws at other scales. In this article, a different model is presented in detail, based on a State Function, with clear advantages, since it involves mechanisms at all levels, respecting the principle of "breaking temporal symmetry", according to irreversible thermodynamics.

I. THE PROBLEM OF CLASSICAL DYNAMICS AND THE NAVIER-STOKES EQUATION

I.1 SOME LIMITATIONS OF THE NAVIER-STOKES APPLICATION

A classic theme in hydraulics and fluid mechanics is the difficulty of understanding and solving problems and calculations related to turbulence flows from nonlinear differential equations, such as the Navier-Stokes equation, to describe the motion of a viscous fluid, which is usually used, with great limitations [1].

$$\rho\left(\frac{\partial v}{\partial t} + v * \nabla v\right) = -\nabla p + \nabla * \tau + f \quad (1)$$

This equation is a developed version of Newton's 2nd law, since the left member is the acceleration of a very small region of fluid moving in the flow, and the right member is the sum of the forces that create that acceleration. Here "ρ" is the density of the liquid, "v" is the velocity of the region, "p" is the pressure, "τ" is the tangential frictional stress on the surface of the region, and "f" is the internal forces of the bodies [2]. Although versions of these equations developed in numerical methods are currently used, their application is not general due to limitations outlined below.

A first problem arises regarding the "smallness" of the region of the fluid under consideration, since it cannot be very large, since the nature of the differential equation used fails, since the concept of "limit" involves smaller and smaller regions; But it cannot be infinitely small, because the fluid is not ideally "continuous" (infinitely divisible and homogeneous), making it difficult to establish how far this equation can represent turbulence, which is composed of increasingly large whirlwinds [3].

A second problem, but not the least difficult, is that its mathematical nature is non-linear, which implies an analytical difficulty, since there are currently no general methods for its resolution, and because in the conceptual part, this mechanism is based on feedback loops in which cause, and effect are intermingled. According to [4],[5] making It virtually impossible to separate them.

A third problem, equally significant, is that the new dynamics of irreversible processes [6],[7] postulate that the "trajectories" of bodies in the space of phases collapse into massive irreversible systems. This effect derives from the interpretation of mechanical interactions as coupling "degrees of freedom" associated with Fourier frequencies [8].

In these cases, the interaction of these degrees of freedom can become "resonant", implying "very small" divisors, which lead to the destruction of the trajectories considered, which tend to infinity. Figure 1.

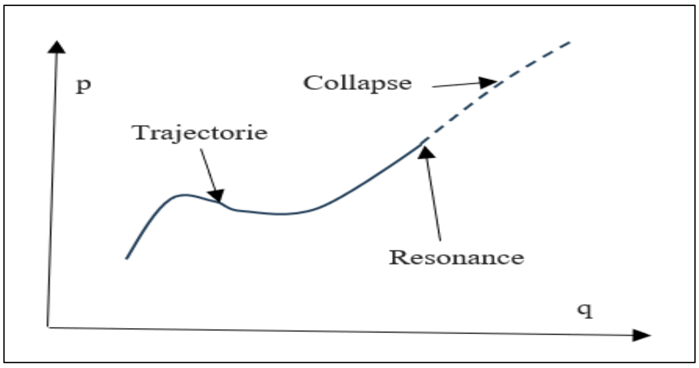

Figure 1: Trajectories in the space of phases.
Source: Authors, (2024).

While it is true that the mechanical description of motions in a turbulent fluid is impossible in terms of trajectory, it is possible to define it in statistical terms, by means of "probability distributions", ρ, which are the only elements of physical description that survive the absence of "computability" in these chaotic systems.

This paradigm shift is not minor because the dynamics of "distributions" are oriented in time, microscopically coinciding with the rupture of the temporal symmetry that is observed macroscopically.

I.2 OPEN SYSTEMS AND PROCESSES OF DYNAMIC EQUILIBRIUM.

Real physical bodies are open systems in that they receive and expel energy and substance from and outward. Depending on this balance, an open system, close to equilibrium, promotes processes of "Dynamic Equilibrium" in which the system achieves "stable states" of quasi-equilibrium in Figure 2.

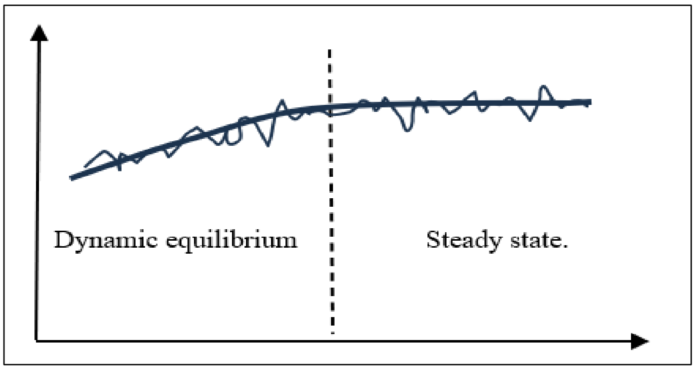

Figure 2: Open systems seeking a "stable state" with quasi-equilibrium.
Source: Authors, (2024).

Small variations in state variables are random distributions of a Markovian nature, in that they represent a drift towards more probable states, i.e., their mean value evolving with an "arrow of time".[9][10]. The representation of this mean value is a deterministic "pattern" of the evolution of these real systems, and together with the fluctuations, they represent a single facet of the process of "Dynamic Equilibrium" in Figure 3.

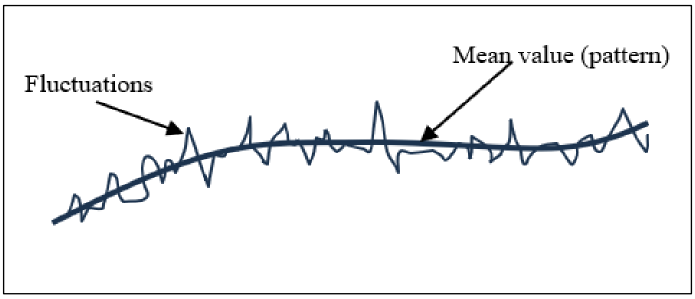

Figure 3: Fluctuations and mean value as an indivisible part of the process of "Dynamic Equilibrium".
Source: Authors, (2024).

As long as the Navier-Stokes equation corresponds to some degree to the type of motion described by "trajectories," and its application does not correspond to an irreversible, time-oriented process, its application to the practical problems of fluid mechanics will be very limited.

II. CONDICIONES PARA UNA DESCRIPCION COMPLETA EN DINAMICA FLUVIAL

II. 1. MORE IS DIFFERENT", A NEW PARADIGM IN MODERN SCIENCE

The "intelligibility" of nature, according to the philosopher A.N. Whitehead, [11] results as a product of a system of general ideas, which is necessary, logical, coherent, in which the function of all the elements of experience can be interpreted.

In this context, the centuries-old scientific paradigm of "Reductionism" had remained unquestioned, [12] in which understanding the world was possible only by understanding its most basic parts. For example, by understanding atoms, it was possible to understand chemistry and hence the very essence of life.

This fallacy, which ignored some difficulties inherent in contemporary science, was brought to the fore by Nobel laureate A.P. Anderson in 1973 with his hypothesis: "More is different", [13],[14] in which, due to the breaking of temporal symmetry, a new hierarchy appears in nature, an "Emergence" of new physical facts at each level of reality.

Thus, for example, even if the balance of forces and acceleration effects in a very small plot of a fluid under turbulence conditions were described in great detail, it was not possible to anticipate feedback mechanisms, which appearing at a second level, not explicitly described in the Navier-Stokes equation, would greatly disturb the fidelity of the model in relation to experimental reality.

II.2. "STEADY STATES" FOR OPEN RIVER SYSTEMS

In an isolated system (without any interaction with the outside), over time, the energy is distributed homogeneously throughout the system, reaching a limiting thermal equilibrium (single, equal temperature at all points of the system). This limit equilibrium is characterized by a maximum of entropy. (***S→max***).

In a closed system (which only exchanges energy with the environment, but no substance), these exchanges of energy with the outside can define a given equilibrium, in which the temperature no longer varies (***T≈ Cte).***

$$F \approx E - T * S \tag{2}$$

This system varies between the predominance of an "energetic" scheme (E>T*S), or an entropic scheme (T*S>E). At low temperatures, an "orderly" system (Crystal like) predominates, while at medium temperature there is a mixture of order and disorder (liquid like), and at high temperature "disorder" (gas like) predominates.

In an open system (which exchanges energy and substance with the environment) different components of entropy must be identified:

$$dS \approx diS + deS \quad (3)$$

The total entropy, S, is the algebraic sum of the entropy produced within the system by irreversible (loss) processes, diS, and of the entropy entering or leaving the system, deS, from and out in Figure 4.

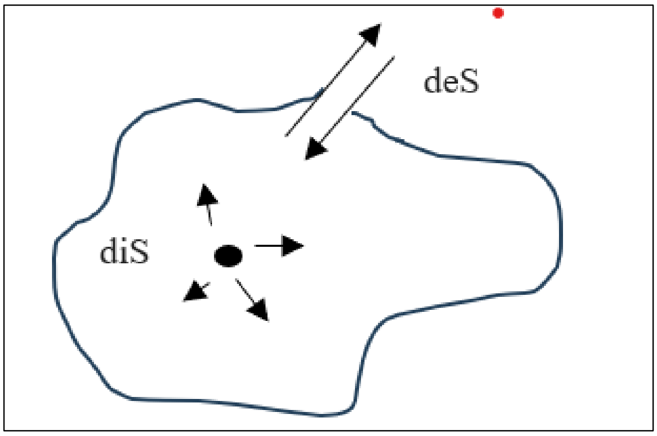

Figure 4: Open systems.
Source: Authors, (2024).

In this case the equilibrium is a steady state that corresponds to the Minimum Entropy Output", P≈ diS/dT, although the total entropy, S, is also a relative maximum. Depending on the balance between the internal and the external, the production of entropy and entropy will have larger or smaller sizes (dotted line) in Figure 5.

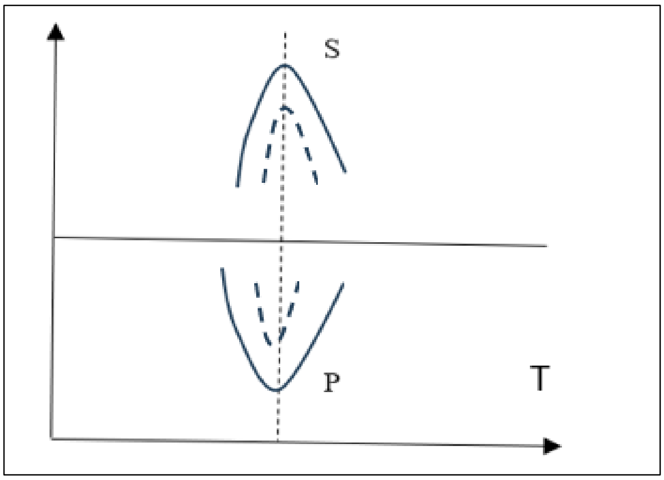

Figure 5: Open systems.
Source: Authors, (2024).

River systems are, of course, open systems, in which entropy maximums will be presented, depending on the balance between what is produced inside and what is exchanged with the outside. The most direct manifestation of the entropy condition in this type of system is the so-called "Granulation Volume", or "Coarse Grain Volume", which is defined by the Boltzmann equation:

$$S \approx k * Ln\,(W) \approx k * Ln(V) \quad (4)$$

Here k is Boltzmann's constant, W is the number of microstates indistinguishable from the energy of the system, and V is the volume of the macrostate, which is observed with an instrument. This volume is identified as the "Coarse Grain" which defines the stable state of the system. For maximum entropy you will have a certain volume of "coarse granules". In the riverbed, the size of these granulation zones will be larger or smaller, depending on the balance, in the channel under consideration. Figure 6 [15],[16].

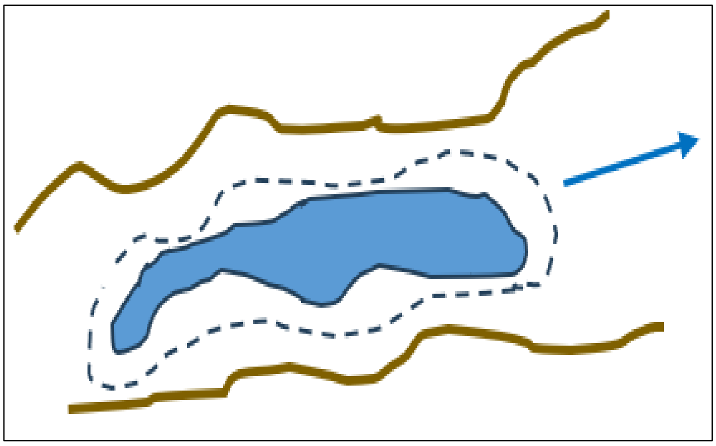

Figure 6: Variable granulation zones depending on the balance.
Source: Authors, (2024).

II.3 "LINEAR" REGION FOR IRREVERSIBLE THERMODYNAMICS APPROXIMATION FOR NATURAL FLOWS, AND INTERMITECIES DUE TO LACK OF PERFEC "LINEAR" REGIME.

"Linear" regime is defined as the thermodynamic region close to equilibrium [17], region in which thermodynamic forces and flows are proportional to each other. A thermodynamic force is basically a gradient of gravitational, electrical, or thermal potential, which originates a flow of charge, mass, electric charge, or heat. In this regime the forces are weak, and so the flows [18].

When you have this regime, the entropy production is minimal, and the entropy is maximum, i.e., the volume of "coarse grain" is maximum, compatible with the balance at the boundary of the system. Applying the criterion of proportionality between thermodynamic flow and force for a natural flow, [19] it is approximately necessary that:

$$U \sim \frac{dH}{dX} \approx S \quad (5)$$

But according to the Chezy-Manning equation, the velocity is proportional to the square root of the Slope.

$$U \sim \sqrt{S} \quad (6)$$

An analysis of the "non-linearity" of the above equation, for specific typical cases, yields percentages of approximately 15%, using Taylor-MacClaurin serial analysis [20]. Now, the net effect of not complying with a perfect "linearity" is that the coarse granularity zone is not perfect either, in the sense that it has randomly distributed "patches" that correspond to different macro states, which are detected as "intermittencies" with the instruments. Figure 7.

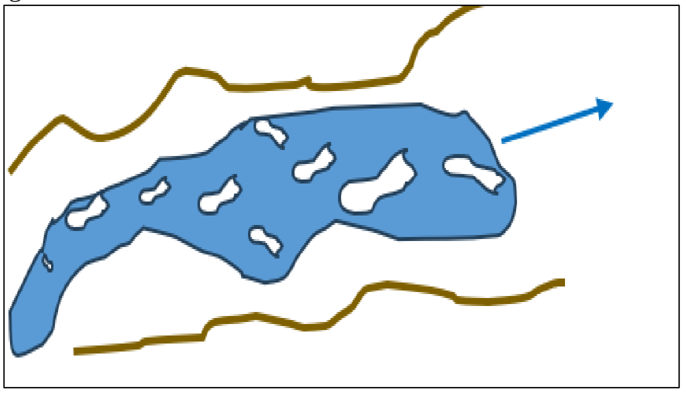

Figure 7: Intermittency of coarse granules.
Source: Authors, (2024).

These intermittencies are reflected as fluctuations in the measurements of the state parameters, characterized by larger "variances".

III. CONDITIONS FOR AN ALTERNATIVE SOLUTION TO RIVER DYNAMICS: A DISCUSSION ON NATURE OF STATISTICAL DISTRIBUTIONS.

III.1 MARKOVIAN MODEL FOR DESIRED DISTRIBUTION.

A proposal for a successful methodology that can replace the Navier-Stokes equation in contemporary hydrometry must meet the requirements that Whitehead has imposed on the method that contains generality, logical coherence, and correspondence with all aspects of experimental making.

The first thing to analyze is the general nature of the type of probabilistic distribution to be considered, its nature and the consequences of its application to river dynamics, in particular it is necessary to establish two basic ideas on which to build the desired model:

The model must respond to the principle of the "Detailed Balance", in such a way that the transitions, from left or rigth, of the most probable value- mean value-are equivalent (Gaussian model).

B.- That the Distribution responds to the fact that the macroscopic observation tends to its most probable value, and that the fluctuations correspond to localized events, close to that value, and have very small magnitudes and times, with random occurrence.

These conditions are fulfilled by the so-called Markov Process, in which the "memory" of the successive transitions is lost, except for the last one. This probabilistic process, although totally random, shows an evolution in only one temporal direction. Since the actual processes in the "linear" region of irreversible thermodynamics are dissipative and if they correspond to a Markovian process, it must be compatible with the increase of entropy to the stationary value, in Figure 5 [21],[22]. In this case, if the entropy reaches a relative maximum peak, $dS \approx 0$, then $d_iS \approx -d_eS$, and the system transfers entropy to the outside.

If we now reinterpret Figure 3, which shows the evolution of an open system (the turbulent flow) towards the "steady state", allowed by the thermodynamic (energy and substance) bonds with the outside, we can draw the following, showing how the statistical evolution of the dynamics is applied to the turbulent flow in Figure 8.

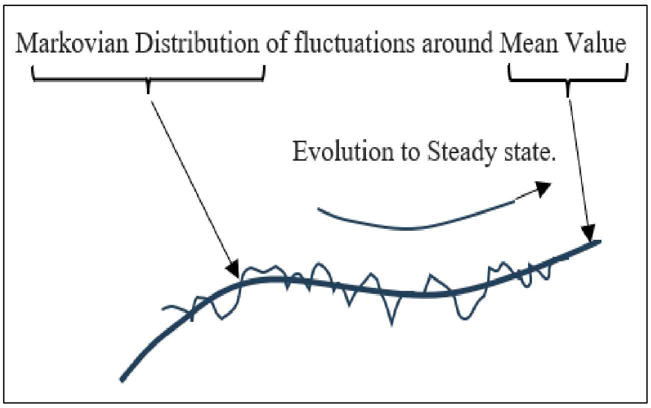

Figure 8: Nature of evolution toward "Steady state" in a turbulent flow.
Source: Authors, (2024).

III.2 ERGODIC NATURE OF THE MARKOVIAN DISTRIBUTION IN FLOW.

If we consider that a Markovian process is basically a Gaussian one, essentially linked to the theorem of the Central Limit, and that this process is prevalent in many fields of physics, important theorems of linear transformation have been defined [23]. One of which is the Ergodic theorem, in which the time function linked to evolution has, for long times, the same spatio-temporal statistical characteristic for various "samples" (measurements) that are taken.

Figure 9 shows an ergodic "band" in which the measurements will have a similar value, in "Steady state" condition. The definition of this band depends on the "mathematical expectation" of the stationary random function remaining constant for different measurements in that band, which occurs for sufficiently long times in which the randomness of the distribution is manifested [24].

An example of this situation is Prandtl's lateral velocity distribution, which is almost flat, along the width of the river [25].

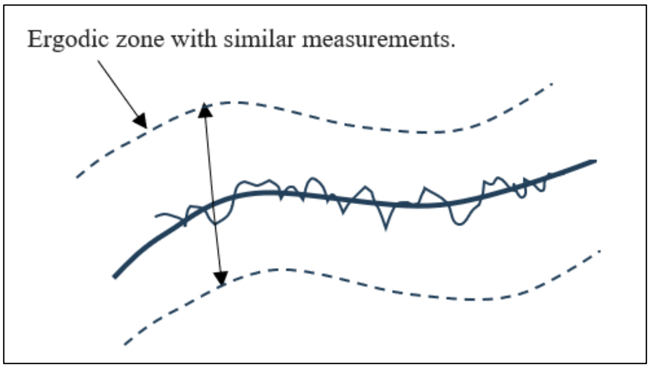

Figure 9: Ergodic zone with similar value of measurements.
Source: Authors, (2024).

III.3 ANALISIS DE FUNCIONES DE ESTADO COMO UNA POSIBLE SOLUCION

State functions are mathematical entities that connect states of a system by varying their state parameters, which are variables that depend only on their "internal physical condition" in relation to the process, defining it univocally, and that does not depend on any concept of microscopic structure [26],[27]. Its mathematical definition, in a flow for example, $\Phi(U,E,t)$ is based on the fact that if an open system that evolves in a turbulent flow has as state parameters: U, E, and also evolves over time, a total differential is configured, as follows:

$$U = g(\Phi), \ E = w(\Phi) \ and \ t = h(\Phi) \quad (7)$$

and:

$$d\Phi = \left(\frac{\partial \Phi}{\partial U}\right) dU + \left(\frac{\partial \Phi}{\partial E}\right) dE + \left(\frac{\partial \Phi}{\partial t}\right) dt \quad (8)$$

This expression indicates that the state function, Φ(U,E,t), depends only on the start and end points, i.e. it is a "point" function, not a "process" function, which depends on the path followed, according to the Schwartz condition [28].

$$\oint d\Phi = 0 \quad (9)$$

Now, the fact that state functions depend only on start (ΦA) and end points (ΦB) excludes the problem of collapsing trajectories, which are "process" events, essentially in Figure 10.

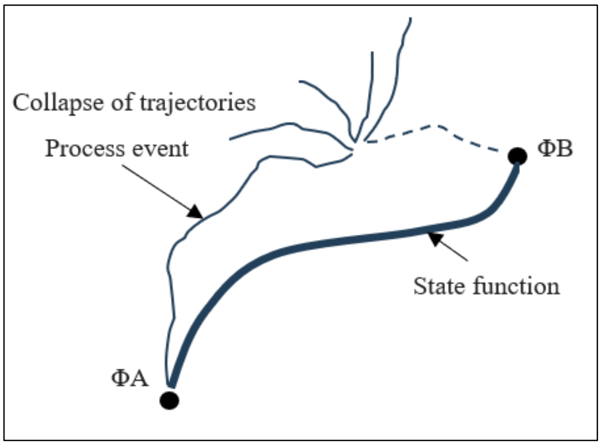

Figure 10. The state function excludes collapsed paths (process events).
Source: Authors, (2024).

On the other hand, this state function is associated as an average value with the Markovian distribution in Figure 8 and respects the basic fact that it is time oriented.

IV. A STATE FUNCTION THAT GUIDES THE EVOLUTION OF THE FLOW IN TURBULENCE.

It will be shown in this section that although tracers are used as marker substances to measure movement with instruments, in reality the state function that will be defined will describe in its final segment the evolution of turbulence itself.

IV.1 THE EVOLUTION OF STATE FUNCTION: HOW TO DEFINE AND APPLY IT

The authors have presented an equation for the average flow velocity, based on dispersive forces of an electrical nature (Van der Waals), with a structure similar to the classical Chezy-Manning equation, based on mechanical interactions. [29].

$$U \approx \frac{1}{\Phi}\sqrt{\frac{2*E}{\tau}} \quad (10)$$

and:

$$U \approx \frac{R^{\frac{2}{3}}}{n}\sqrt{S} \quad (11)$$

In the unidirectional equation (10), E is the longitudinal coefficient of dispersion, and "τ" is a characteristic time of the self-replicating process, and is worth:

´

$$\frac{t}{\tau} \approx \delta \approx 4.6692 \quad (12)$$

Here "δ" is the chaotic period doubling, and it is actually the hallmark of the Gaussian nature of the process, since this remarkable number, initially discovered by [30], derives from the mean value of Brownian dynamics, studied at the beginning of the 20th century by [31] but placed within a unimodal scheme of the natural growth model (logistic curve). Svedberg found that the mean value of the distribution of colloidal particles in fluid media (ultracentrifuge) was described by the Poisson model, with an average concentration value <C(t)>≈1.54. Therefore, the growth rate in the logistics curve, for Brownian distribution, is:

$$r(t) \approx e^{\frac{1}{T}*\int_0^T C(t)dt} \approx e^{1.54} \approx 4.6692 \quad (13)$$

Since this logistic curve is a universal model of growth, it of course applies to the self-similarity conformation of Brownian motion and is thus the basis of equation (12).

$$r(t) \approx \frac{t}{\tau} \approx 4.6692 \quad (14)$$

Ahora, volviendo a la descripción de la aplicación de la función de estado al caso fluvial, for classical equation (11), "R" is the hydraulic radius, "n" is the Roughness coefficient, and "S" is the Slope of the flow. If we equate the two definitions of speed, we have that:

$$\Phi \approx \frac{n}{R^{\frac{2}{3}}}\sqrt{\frac{2E}{S}} \quad (15)$$

In this way, the geometric and geomorphological parameters can be set as a function of the State Function, i.e., we have a complete fluvial description from Φ(U,E,t).
The state function can be approximately defined at ***tp*** (peak time) as: [32].

$$\Phi \approx \frac{M}{Q*\gamma*1.16} * \frac{1}{\sqrt[3]{tp}} \quad (16)$$

Here, ***M*** is the mass of tracer injected, ***tp*** is the peak time of tracer curve, and ***γ*** is a characteristic parameter of each solute used, and which must be calculated experimentally, with "***Cp***" as peak concentration of tracer in measurement point.:

$$\gamma \approx \frac{Cp(tp)}{tp^{-\frac{2}{3}}} \quad (17)$$

Table 1 shows below approximate typical values of "γ" for different types of tracers and different streams.

Table 1: Typical (approx.) values of γ for diverse kind of tracers and streams. Source: Authors.

Tracer	Condition of stream	"γ" Approx. Mean value
'Radioactive (Cl3Au-198)	Q≈8.2 m3/s, L≈1.7 Km	<9351>
Fluorescein	Q≈ 97 L/s, L≈43 m	<123>
RWT	Small Creek, Q≈40 L/s., L≈23 m	<2162>
Saline (NaCl)	Small Creek, Q≈40 L/s., L≈23 m	<1301>

Source: Authors, (2024).

IV.2 THE NATURE OF STATE FUNCTION: ITS RELATIONSHIP WITH THE PHYSICO-CHEMICAL PROCESSES THAT OCCUR TO THE SOLUTE IN THE FLOW

Once the solute is suddenly injected into the flow there is an interaction between the water (dipoles) and the ions of the compound, then the solute diffuses and there is an interaction between the ions of the latter, this occurs until it can be considered a gas, which is considered very rarefied, which occurs when ***Φ≈0.38***, the point at which all the mass is homogeneously "available" in the cross-section of the tracer flow tube.

At this point, the tracer is considered to have all but lost its identity, and the instrument's measurements actually capture the turbulent motion of the water. The dispersion (diffusion) being measured, which originally corresponded to the tracer, in this last phase now corresponds to the mixing movements (self-diffusion) of the water in turbulence [33] in Figure 11.

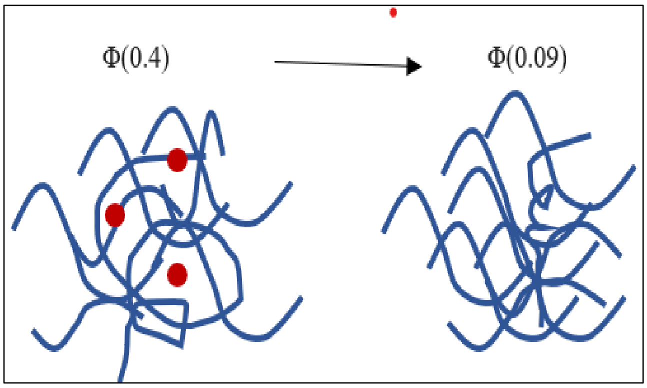

Figure 11: State function measuring turbulence, with or without tracer particles.
Source: Authors, (2024).

Then, it can be established that ***Φ(t)*** evolves as shown in Figure 11.

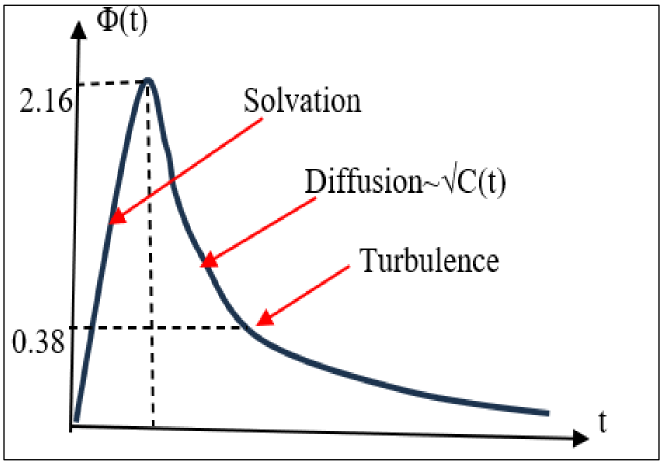

Figure 12: Evolution of State function Φ(t).
Source: Authors, (2024).

The steep segment of curve (up to ***Φ≈ 2.16***) corresponds to the "solvation" process of the solute. The smooth decaying segment corresponds to the evolution of the ions, which diffuse until at Φ≈ 0.38, almost all of them are in the gas phase, and ***Φ(t)*** is considered to describe the turbulent evolution of the flow itself in Figure 13.

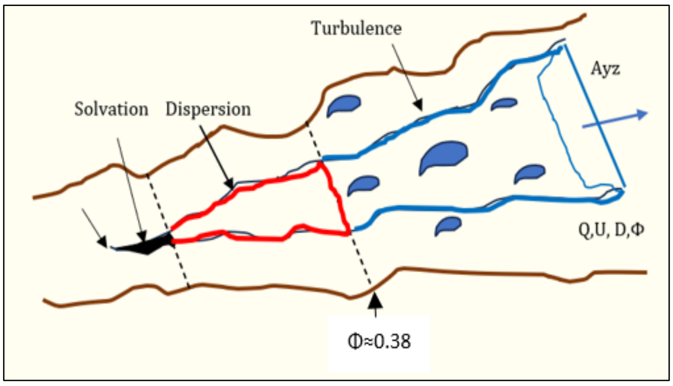

Figure 13: urbulence described by a State functionΦ(t).
Source: Authors, (2024).

This aspects is of great importance for fluvial engineering, because with this method it is possible to estimate values of state parameters at distant sites in the flow, without having to inject large amounts of tracer with a serious threat to the environment of the ecosystem. Figure 14.

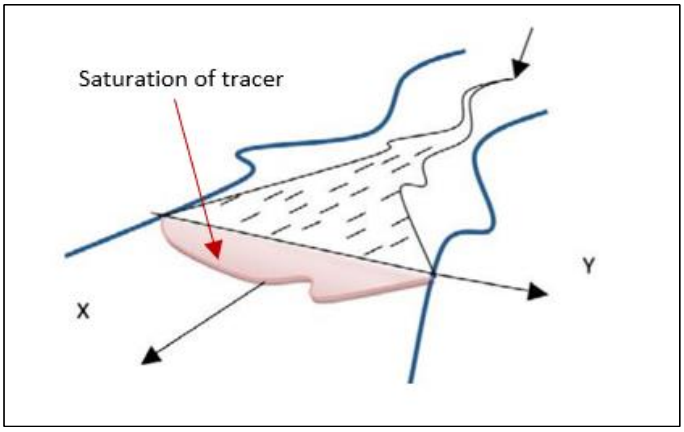

Figure 14. Saturation of tracer in state-of-the art methods.
Source: Authors, (2024).

IV.3 PLANNING FLUVIAL EXPERIMENTS USING THE STATE FUNCTION METHOD

The application of the State Function method to fluvial research allows not only to obtain information with greater scientific content, but also allows the experimenter to simplify monitoring tasks as he must use much less tracer mass, with the consequent environmental preservation. Monitoring is primarily based on selecting a representative section of the channel and, once the conservative solute has been injected, making measurements at two sequential sites separated by a distance proportional to the width of the flow. With these two fluorescent tracer curves (Fluorescein or Rhodamine WT) the two values of ***Φ(t)*** are established and the projection can then be made at much greater distances, if we start from the assumption that this section of the flow is at "Dynamic equilibrium". This condition is approximately verified by the observations of flow and slope constancy. Here "***Pi***" is the point of injection of the solute, "***E1***" and "***E2***" are the two points of the experimental measurement with fluorometers, and "***Pm***" is the point of projection of the value of ***Φm(t),*** based on the two values of this function in the previous points. Based on this data, it is possible to obtain all the information of interest for hydrometry or modelling in Figure 15.

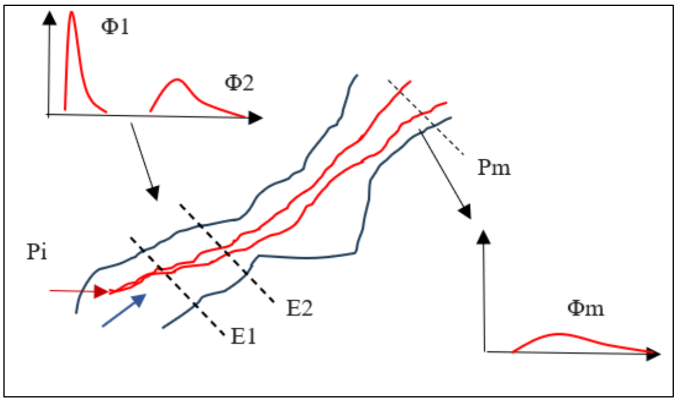

Figure 15: Two tracer measurements to obtain a sequence of State function Φ(t).
Source: Authors, (2024).

It should be noted that it is not necessary to make "injections at the center", as is the optimal requirement in classical methodologies, since the State Function reflects the objective conditions of the injection, including longitudinal and cross-sectional variables [34].

V. CONCLUSIONS

1.- The basic characteristics of natural channels are established, as open systems, evolving in the condition of "Dynamic Equilibrium". This condition is established as resulting from fluctuations and a mean value, which have statistical characteristics compatible with the Gaussian nature and the principle of temporal symmetry breaking. (Markovian distribution).

2.-The description of phenomena as complex as turbulent flow requires the concurrence of multiple concepts, in such a way that the description is as complete as possible. The current level of knowledge of hydrodynamics is limited by the fact that its theoretical approach is usually limited to the representation of the evolution of phenomena by means of non-linear differential equations, which give only "local" information, which is difficult to interpret and handle analytically, because it is a very difficult to interpret and manipulate analytically, because it is a "reductionist" theory, limited only to the most basic levels of reality.

3.- Opposing, "emerging" models, which draw attention to the impossibility of solving problems of interpretation at various scales, must be based on "non-local" mechanisms, which include different but congruent parts of reality, as A.N. Whitehead demands.

4.- The description of the evolution of hydrodynamics is traditionally done by means of a "reductionist" theory of the "Navier-Stokes" type, which shares the limitations and difficulties of this type of model. A model that solves these problems must be a "non-local" one, which also involves the developments of irreversible thermodynamics, close to equilibrium.

5.- The authors present here an "emergent" model of the "State Function" type, which eliminates the problems identified in the resolution of hydrodynamic designs with existing theories.

VI. AUTHOR'S CONTRIBUTION

Conceptualization: Alfredo Constain A and Julian Ramos S.
Methodology: Alfredo Constain A and Julian Ramos S.
Investigation: Alfredo Constain A and Julian Ramos S.
Discussion of results: Alfredo Constain A and Julian Ramos S.
Writing – Original Draft: Alfredo Constain A and Julian Ramos S.
Writing – Review and Editing: Alfredo Constain A and Julian Ramos S.
Resources: Alfredo Constain A and Julian Ramos S.
Supervision: Alfredo Constain A and Julian Ramos S.
Approval of the final text: Alfredo Constain A and Julian Ramos S.

VII. REFERENCES

[1] Stewart I. "¿Juega Dios a los dados?". Grijalbo-Mondadori. Barcelona, 1996.

[2] Stewart I. "17 ecuaciones que cambiaron el mundo". Critica, Barcelona. 2013.

[3] Stewart I. Ibid, 1996.

[4] Annila A. & Makela T. Natural patterns of energy dispersal. Physics of life rev. 7PP 477-498. 2010.

[5]-Annila A.& Ketto J. The capricious character of nature. Life ,ISSN 2075. 2012.

[6] Prigogine I. ¿Tan solo una ilusión?, Matema- Tusquets, Barcelona. 1997.

[7] Kondepudi D.& Prigogine I. Modern thermodynamics. From heat engines to dissipative structures. Wiley. New York., 1999.

[8] Prigogine I. El fin de las certidumbres. Aguilar. Bogotá. 2001.

[9] Prigogine I. Las leyes del caos. Critica, Barcelona. 1999.

[10] Nicolis G. & Prigogine I. La estructura de lo complejo. Alianza. Editorial. Madrid. 1997.

[11] Prigogine I. Ibid. 2001.

[12] Miller J. H. A crude look at the whole. Basic Books, New York, 2015.

[13] Anderson P.W. Moore is different. Science. Vol. 117. 1972.

[14] Anderson P.W. Basic notions of condensed matter physics. Amazon. Finley Editor, Princeton. 2018.

[15] Langbein W.B. The hydraulic geometry of shallow estuary. Hyd. Sci. JOU. 8: 3, pp 84-94. 2009.

[16] Langbein W. & Leopold L. Quasi-equilibrium states in channel morphology. USGS. Vol 262. P782-794. 1964.

[17] Kondepudi D.& Prigogine I. Ibid. 1999.

[18] Prigogine I. Ibid. 1997.

[19] Constain A. The svedberg number 1.54, as the basis of a State function describing the evolution of turbulence and dispersion. Intech Open, chapter of book. London. 2024..

[20] Constain A. Ibid. 2024.

[21] Nicolis G. & Prigogine I. Ibid. 1997.

[22] Pugachev. V.S. Introducción a la teoría de las probabilidades. Editorial Mir. Moscú, 1973.

[23] Nicolis G. & Prigogine I. Ibid. 1997.

[24] Pugachev. V.S. Ibid. 1973.

[25] Nekrasov. B. Hidraulica. Editorial Mir, Moscú, 1968.

[26] Planck M. Treatise on thermodynamics. Dover. 1955.

[27] Kirillin V.A., Sichev V.V. & Sheindlin A. Termodinámica técnica., Editorial Mir, Moscú. 1986.
'
[28] Morowitz. H. Entropia para biólogos. Blume, Madrid. 1975.

[29] Constain A. Definición y análisis de una función de evolución de solutos dispersivos en flujos naturales. Dyna. No. 175. Medellín, 2012.

[30] Gleick J. Caos: La creación de una ciencia. Critica. Barcelona. 2012.

[31] Constain A. Ibid. 2024.

[32] Constain A. Ibid. 2024.

[33] Frenkel J. Kinetic theory of liquids. Dover. 1955.

[34] Constain A. Ibid. 2024.