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RESEARCH ARTICLE OPEN ACCESS

SPRINGBACK TRENDS WITH DIFFERENT PROCESS PARAMETERS OF THREE SELECTED METALS

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ABSTRACT

Products or components manufactured by the sheet metal process are indispensable in this contemporary era, from daily-use metal jars to high-tech air vehicles. Nonetheless, there are a variety of sheet metal manufacturing processes for fabricating these goods; the air-bending sheet metal process is one of the most commonly conducted methods among those. After air bending, commonly used metals — Mild Steel, Aluminum, and Stainless Steel — tend to demonstrate an unwanted characteristic called springback, which requires being controlled to produce extremely precise parts. Therefore, this study aimed to develop an empirical model that would improve springback prediction, which would ultimately assist in minimizing the springback effect in bending operations. Moreover, exploring springback trends with process-related parameters was also the goal of this work. To achieve objectives thoroughly, several methodological approaches were incorporated sequentially: Box-Behnken experimental design, experimental data collection, conducting an ANOVA test, empirical model selection, model development, and model validation. Finally. in the validation phase, it was found that the developed linear models were able to prognosis springback with acceptable errors. From that mathematical model, springback trends with process-related parameters were explored. Overall, this study provides a wider view for observing springback trends with different factors.

I. INTRODUCTION

Products or components manufactured by sheet metal are indispensable in this contemporary era, from daily-used metal jars to high-tech air vehicles, including automobile panels, airplane skins, cans for beverages, and frames for electronic devices [1]. As a result, the sheet metal process is incorporated in many industries — especially in aircraft, automotive, food, and home appliances [2]. These industries favor sheet metal fabrication, particularly the air-bending process, for its many significant [3]. Firstly, air-bending is extremely inexpensive and versatile, as it permits the fabrication of a vast range of bend angle configurations to be formed with the same set of instruments [4] [5]. Secondly, the air-bending method gives the flexibility required for manufacturing numbers. For instance, it may gratify the need to create decent parts even with a batch size of just one [5]. Thirdly, it is appropriate for sheet components with complicated, curved faces [6].

Fourthly, it requires a lower bending force than other bending modes, such as coining and bottoming [7]. Therefore, there are several significant air-bending sheet metal processes in manufacturing industries. After understanding the significance of the air-bending sheet metal process from the aforementioned paragraph, the readers of this article may be curious to know what air-bending exactly means. Air-bending is a major conventional sheet-metal bending technique [8] [9], which involves using a punch tip to press a piece of metal — both thin and thick [10] — into a die, while leaving air between the metal and the die. This process is also known as the three-point bending process [4], because the die touches the two points of the specimen, and the punch tip touches the specimen in the middle with one more point. Like other bending processes, the air-bending technique has one crucial undesired feature, which is springback [11] defined as an elastically driven shift in the shape of a component after forming [12]. In other words, springback happens, technically speaking, in metal forming due to the primarily elastic recovery of material following the removal of the punch force [13] [14]. Since the bottoming is nonexistent in air-bending, the amount of springback is high for this particular process [4]. As a consequence, this springback may produce substantial shape deformities in the following natural shaping stage. It is therefore vital to understand which elements affect the springback, to estimate the amount of the springback [15], and prevent defects. To understand springback-affecting factors, there were many springback research works carried out by several scholars, especially for the air-bending process. One background work on air-bending in DP600 sheet material by Ozdemir showed that as the punch tip radius rose, the springback value increased; on the other hand, when the sheet thickness increased, the springback value reduced [16].

In addition to that, it was discovered by Gupta et al. during air-bending of electro-galvanized CR4 steel that springback increased with the increase in the width of the die for every punch journey — where the flexible die width was varied, but die radius, punch radius, and punch speed were maintained the same [17]. One more study by Thipprakmas et al. explored the influence of process parameters on springback in V-bending processes, by utilizing the finite element method. The findings demonstrated that material thickness had a large impact on springback and experimental validation revealed excellent agreement between finite element simulation and real findings [18]. Another finite element method-based research along experimental test was done by Xie, where it was demonstrated that springback in the air bending process rose with the punch displacement, punch radius, and die spread increased [19]. Further, Garcia-Romeu et al. depicted that springback was related to bending angle and sheet thickness. It was also noted that the bigger the yielding stress was, the bigger the springback amount was [20]. Furthermore, Buang et al. found that higher yield stress, caused a larger springback; the springback decreased with Young's modulus increased; and thicker materials had less springback [21].

From the literature, it is clear that springback depends on numerous factors, some of them are process parameters and others are mechanical properties. Although literature works were successful in identifying affecting springbacks factors, some research gaps were present. For instance, one research gap was in finding the springback sensitivity for process parameters. In addition, though few papers were found where a mathematical equation was developed for process parameters, no paper was found where a mathematical model had both process parameters and categorial material variable — process parameters could have provided process-related trends, on the other hand, the categorical material variable would have revealed material properties related trends. It can be assumed that the reason behind this could be the difficulty of blending the mechanical properties of metals with the process parameters. However, the curve-fitting technique may be helpful in combining the mechanical properties of the specimen and process parameters to engender a mathematical model.

Thus, research gaps existed for sensitivity analysis, process trend analysis, and material properties trend analysis. To address the aforementioned research gaps, this study had some objectives. The first objective was to develop an empirical mathematical model for prognosis springback in light of a curve-fitting equation. This equation included three process parameters along with one categorical variable namely material. So, this mathematical model might be useful to predict the springback more accurately. After the development of the mathematical model, springback sensitivity could be observed from the same mathematical model. Thus, the second goal of this work was to order the springback sensitivity from highest to lowest for process matrices. The third purpose of this study was to seek trends of springback with process parameters. The rest of this paper is organized as follows: ***Section 2*** shows methods; ***Section 3*** depicts analyses and findings; ***Section 4*** illustrates discussion; and finally, ***Section 5*** presents a conclusion.

II. MATERIALS AND METHODS

This study required many things to follow sequentially and all of them are visually illustrated by a flowchart in ***Fig. 1***. Hopefully, this flowchart makes the study's methods demystify for potential readers.

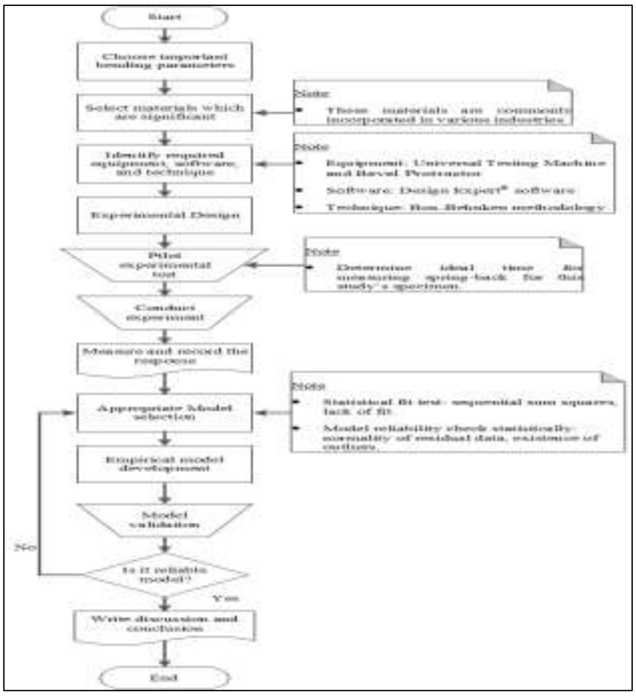

Figure 1: Sequences of this study.
Source: Authors, (2025).

II.1 STUDY'S FACTORS SPECIFYING

There are tremendous parameters that can influence springback [22]. In this study, a total of four investigational parameters were taken. Three of them were process parameters: initial bend angle, punch radius to sheet thickness ratio (P.R/S.T), and die gap. These three process parameters were likely to reveal springback trends with process parameters. On the other hand, one categorical factor — material was going to provide implicit for springback trends with mechanical properties. After establishing a mathematical model, different materials could be compared with the springback trends. Since different materials have different particular mechanical properties, thus, the mechanical properties related to springback trends could be revealed.

II.2 MATERIALS

An appropriate material section is a critical criterion for pursuing research pertinent to springback characteristics. Commonly employed industrial metals were given priority for this study. For example, Mild Steel S355 grade is one of the widely used materials for important structural applications [23]. Another vital material that could be considered for this research was Stainless Steel 304 — a commonly used metal in different kinds of industries for several purposes [24]. One more useful material is the Aluminum 7050 alloy, particularly in the aerospace industry [25]. Therefore, these three widely used materials were chosen in this research: Mild Steel (S355 grade with 0.20% carbon), Stainless Steel 304 (comprising 18% chromium and 0.11% carbon), and Aluminum 7050 alloy (containing 89% aluminum). Furthermore, it should be mentioned that sheets of these materials were examined at three distinct thicknesses: 1 millimeter, 1.5 millimeters, and 2 millimeters. It will be beneficial to get the springback trend with sheet thickness. Moreover, to ensure precision in extracting the desired size from the bulk material, each sheet was carefully prepared to the exact dimensions of 150 millimeters in length and 30 millimeters in width, by using a hydraulic cutter.

II.3 NECESSARY TOOLS: UNIVERSAL TESTING MACHINE AND DESIGN EXPERT® SOFTWARE

After choosing the desired materials, proper equipment selection is another indispensable step for any experimental research. In this experiment, a Universal Testing Machine was used. It was incorporated into this study to make the bend of the aforementioned three metals. Figure. 2 delineates the Universal Testing Machine, which supported the experiment. In addition to this, ***Fig. 3*** depicts the photographic view of the experimental setup for air bending of specimens. Another hardware equipment, the Bevel Protector, was utilized in this study to measure the angles.

Figure 2: A real view of the experimental setup under a Universal Testing Machine for air-bending.
Source: Authors, (2025).

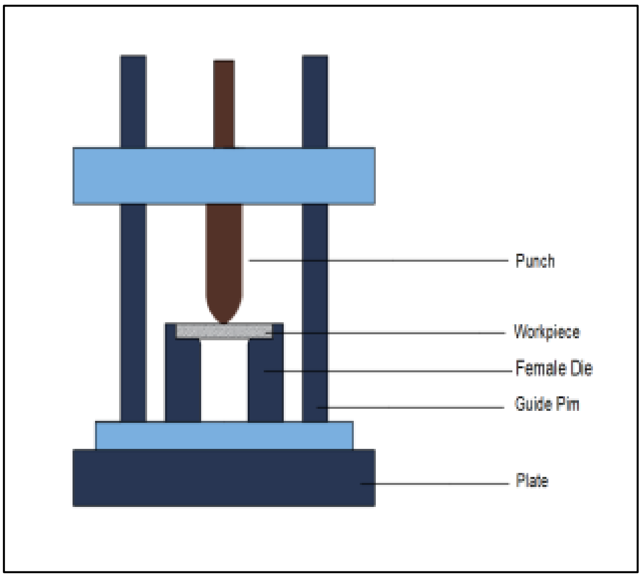

Figure 3**:** A photographic view of the experimental setup for air-bending.
Source: Authors, (2025).

Apart from the hardware tools, a vital software tool for this study was the Design Expert® — developed by State Ease [26]. This software provided the least number of Runs — the number of experiments that must be carried out according to the selected experimental design — required in light of the given factors [26]. In this investigation, there were several variable factors, delineated in ***Table 1***, as well as constant factors, illustrated in *Table 2*, which were inserted into Design Expert® software (version 13).

Table 1: Experimental variables and their corresponding numeric codes.

Process factors	Levels of each factor		
	1	2	3
Die gap	65	70	75
P.R/S.T	5	10	15
Set angle	120	135	150
Material	Mild Steel S355 grade	Aluminum7050 alloy	Stainless Steel 304

Source: Authors, (2025).

Table 2: Four constant factors for one response factor, springback.

Constant factor	***Corresponding value***
Die radius	5 millimeters
Initial punching force	500 Newton
Punch velocity	0.8
Sheet dimension	150 millimeters × 30 millimeters

Source: Authors, (2025).

II.4 RESPONSE SURFACE METHODOLOGY

Response Surface Methodology is a collection of statistical design and numerical optimization techniques used to optimize processes [27]. One sort of Response Surface Methodology is the Box-Behnken Design algorithm, which can generate higher-order response surfaces using fewer required Runs than a normal factorial technique [28]. In this study, by using the Box-Behnken methodology as a design matrix, Design Expert® software demonstrated that fifty-one individual Runs were necessary based on ***Table 1*** and ***Table 2***. Therefore, fifty-one experimental tests were conducted to collect data.

II.5 EXPERIMENTAL TESTING PROCEDURES VIA A UNIVERSAL TESTING MACHINE

Basically, a Universal Testing Machine bends sheet metal by using two adjustable dies, and one punch. In this study, sheet specimens were bent at three predetermined angles — 120, 135, and 150 degrees — throughout the experiments. Three deformed specimens of each three materials are depicted in ***Fig. 4***. It needs to be mentioned that the exact initial angle was calculated and achieved by the following equation ***(1)*** where h stands for punch travel distance, d represents die gap, and the symbol θ_i indicates the initial angle. The rationale behind this equation is to apply the trigonometry formula from the right-angle triangle, illustrated in ***Fig. 5***.

$$\tan\left(\frac{\theta_i}{2}\right) = \frac{\frac{d}{2}}{h} \tag{1}$$

Figure 4: deformed specimens of Stainless Steel 304, Mild Steel S355 grade, and Aluminum 7050 alloy.
Source: Authors, (2025).

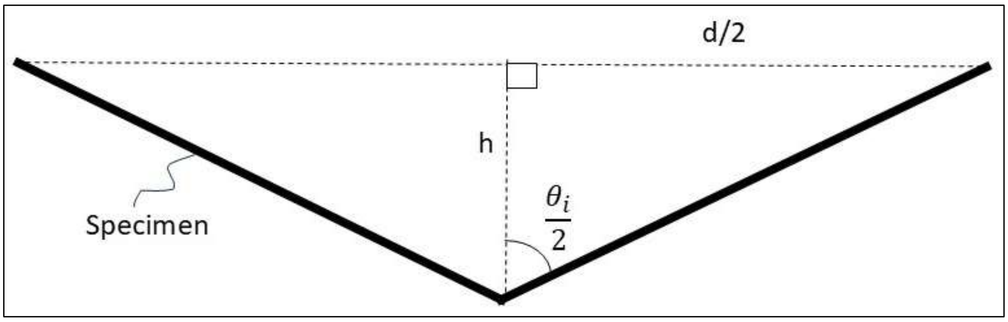

Figure 5: freehand drawing to showcase the logic behind the initial angle calculation.
Source: Authors, (2025).

Afterward, the initial bending angle θ_i was measured and verified instantly by a Bevel Protector. After 32 hours of load removal, the final bending angle θ_f was also measured by a Bevel Protector. Therefore, subtracting the set angle from that measured final bend angle eventually gave the angular magnitude of springback — shown in ***Fig. 6***. Here, equation ***(2),*** represents the angular springback formula [29] [16], where $\Delta\theta$ stands for angular springback.

$$\Delta\theta = \theta_f - \theta_i \tag{2}$$

Where $\theta_f > \theta_i$

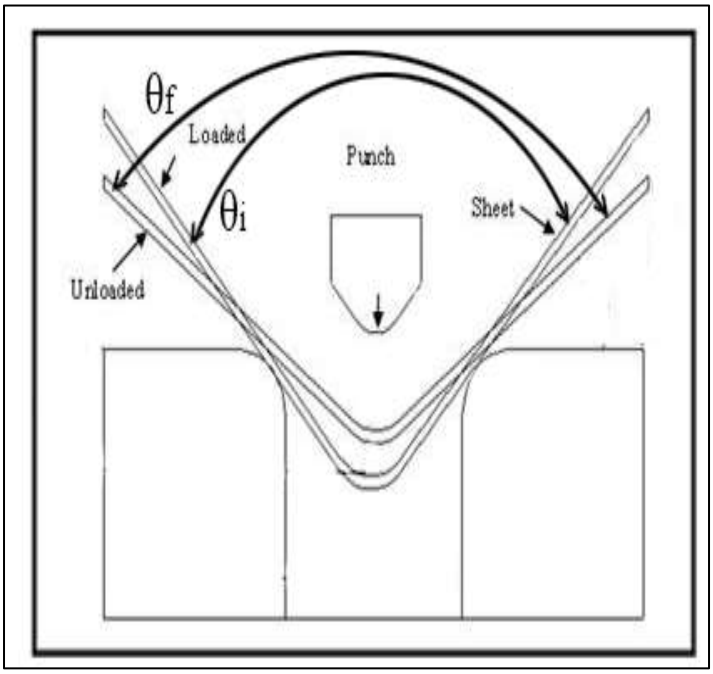

Figure 6: demonstration of initial bend angle and final bend angle by line art.
Source: Authors, (2025).

Additionally, it needs to be mentioned that lubrication during the air-bending process significantly affects both punch force, die, and springback compared to the dry conditions [30]. Hence, lubrication was excluded from the experiment in order to acquire an actual scenario of those factors without affecting the outcomes, therefore, a rational conclusion could be made from this experiment.

II.6 PILOT TEST

In the pilot run, experiments were conducted using all parameters, including die gap, punch radius to sheet thickness ratio, material, and set angle. The result from the pilot run was thoroughly analyzed to detect any anomaly or effect of noise. Apart from this, it needs to be addressed that final bend angles were subsequently measured at 8, 16, 24, 32, and 40 hours of post-bending, in order to observe what the right time interval was for measuring the final springback angle of the parts. Since there was no further springback angle increment after 32 hours; therefore, a decision was made that all springback would be measured after 32 hours during the experimental data collection phase of this study.

II.7 APPROPRIATE MODEL SELECTION

In this phase of the research, an appropriate response model for the response factor was essential for the collected data from the experiment. There are several models, for instance, linear, first-order interaction (2FI), quadratic, and cubic [31]. Selecting the best model among those models involves comparing how well each model fits the data — it is known as the Fit Test from the statistical point of view. To select the right model for the collected data of this study, several statistical techniques were employed. Firstly, a statistical technique namely the *Sequential Model Sum of Squares,* was employed to understand the significance of the model. This technique gave the best model, which had the highest significance. Secondly, another statistical technique, namely the *Lack of Fit Test,* was used to find model inadequacies [32] [33]. Actually, this technique helped to cross-check the model. Put differently, if a model is unable to pass the *Lack of Fit Test,* it indicates that the acquired data was correct. For this test, the P-value of the *Lack of Fit Test* for the model must be more than 0.05 — the accepted criterion.

On the contrary, if the P-value of the *Lack of Fit Test* for the model is less than 0.05, the model is insignificant — the rejected criterion [32]. Thirdly, the performance of different models was also evaluated in light of various metrics such as standard deviation, coefficient of determination (R^2), adjusted R^2, predicted R^2, and the predicted residual error sum of squares (PRESS). These metrics were beneficial in comparing the models and selecting the most appropriate one based on balance performance. Because each of these metrics has statistical importance. For instance, R-squared is a metric that quantifies the proximity of the data points to the regression line that has been fitted [34]. Conversely, the concept behind adjusted R-squared is to consider the inclusion of factors that do not substantially enhance the model. The adjusted R-squared value is always lower than or equal to the R-squared value. All of these three techniques might be called *ANOVA* — it is a widely used set of statistical models aimed at comparing variation between data.

Furthermore, in order to provide statistical reliability for the recherche model, several plots would be considered: the normality plot, scatter diagram, and Box-Cox graph. First, the normal probability plot of residuals is used to assess the normality of the data. If the plotted points align closely with a straight line, the dataset can be considered to be normally distributed [35]. Second, a scatterplot is an effective tool for investigating connections among parameters, discerning relationships beyond facile correlations, and routing more accurate data science practices [36]. Third, the Box-Cox transformation technique is used to enhance the compression of spatial data by promoting normalcy, efficiently lowering spectral dimensions, and eliminating mistakes in spatial values [37] [38]. Thus, if the data need to be transformed from non-normality to normality, the Box-Cox could be utilized. Apart from that, if the data are already normally distributed, the Box-Cox plot will indicate that no transformation is needed. So, it can be a cross-checking technique for reinforcement that the collected data has normality. A question might be raised about how it could be understood that there is no metamorphosis needed. Well, lambda (λ) — a transformation parameter in Box-Cox [39]— values help with this. There are different standard transformations for different lambda values: lambda values of -3, -2, -1, -0.5, 0, 0.5, 1, 2, and 3 represent inverse cubic, inverse square, inverse, inverse root square, logarithmic, root square, no transformation, square, and cubic, respectively [40] [41]. So, a lambda value of one indicates that no conversion is required since the data are already normally distributed.

II.8 EMPIRICAL MODEL DEVELOPMENT

After the selection of a model, it might be time for the development of an empirical model — this kind of model is only supported by experimental data [42]. To put it another way, empirical models are based on correlations obtained from the analysis of experimental data [43]. Empirical models that have been used for curve-fitting processes to generalize the results of experiments [44] [45]. The curve fitting may be achieved by suitable methods to fit polynomials or other functions [44]. It is mandatory to mention that an empirical model can provide trustworthy results when it is based on a substantial amount of test data [46]. In this study, a particular response model was chosen in an earlier phase, and in light of that model, the final empirical model was developed with the help of Design Expert® software..

II.9 RESPONSE SURFACE METHODOLOGY

In experimental validation, the response factor — angular springback — was forecasted in light of process parameters by using the developed empirical model for all three picked distinguished materials. Subsequently, actual angular springback was measured for the same magnitude of those four independent factors. Eventually, errors were calculated by following equation ***(3)***.

$$\textbf{Error}\ (\%) = \frac{\textbf{Actual value} - \textbf{Predicted value}}{\textbf{Actual value}} \times \mathbf{100} \tag{3}$$

III. RESULTS AND DISCUSSIONS

All the phases of this study’s analysis have been sequentially described: experimental data collection, model selection, model development, and finally model validation.

III.1 EXPERIMENTAL DATA COLLECTED VIA USING A UNIVERSAL TESTING MACHINE

All of those recorded specific combinations derived from these experimental fifty-one Runs are detailed in ***Table 3***, where the magnitude of four independent variables and the corresponding value of one dependent variable for each run are shown. These experimental data were imported into Design Expert® (version 13) so that an appropriate response model could be chosen in light of necessary statistical tests.

Table 3: Experimental design matrix of four actual independent process variables with the experimental response of springback.

Std	Run	Factor A: Die gap	Factor B: P.R/S.T	Factor C: Set Angle	Factor D: Material	Response Springback
2	1	75	5	135	Mild Steel	3
40	2	75	10	120	Stainless Steel	10
48	3	70	10	135	Stainless Steel	8
31	4	70	10	135	Aluminum	10
30	5	70	10	135	Aluminum	11
37	6	65	15	135	Stainless Steel	13
32	7	70	10	135	Aluminum	8
29	8	70	15	150	Aluminum	6
10	9	70	15	120	Mild Steel	11
47	10	70	10	135	Stainless Steel	5
36	11	75	5	135	Stainless Steel	6
3	12	65	15	135	Mild Steel	6
46	13	70	15	150	Stainless Steel	7
38	14	75	15	135	Stainless Steel	14
7	15	65	10	150	Mild Steel	5
43	16	70	5	120	Stainless Steel	8
21	17	75	15	135	Aluminum	12
35	18	65	5	135	Stainless Steel	4
45	19	70	5	150	Stainless Steel	5
24	20	65	10	150	Aluminum	9
41	21	65	10	150	Stainless Steel	5
39	22	65	10	120	Stainless Steel	9
44	23	70	15	120	Stainless Steel	10
34	24	70	10	135	Aluminum	9

23	25	75	10	120	Aluminum	11
17	26	70	10	135	Mild Steel	11
50	27	70	10	135	Stainless Steel	5
19	28	75	5	135	Aluminum	7
6	29	75	10	120	Mild Steel	13
11	30	70	5	150	Mild Steel	2
13	31	70	10	135	Mild Steel	7
49	32	70	10	135	Stainless Steel	10
33	33	70	10	135	Aluminum	12
4	34	75	15	135	Mild Steel	6
1	35	65	5	135	Mild Steel	4
12	36	70	15	150	Mild Steel	12
27	37	70	15	120	Aluminum	16
9	38	70	5	120	Mild Steel	5
28	39	70	5	150	Aluminum	3
42	40	75	10	150	Stainless Steel	5
20	41	65	15	135	Aluminum	10
16	42	70	10	135	Mild Steel	9
5	43	65	10	120	Mild Steel	13
25	44	75	10	150	Aluminum	7
8	45	75	10	150	Mild Steel	11
26	46	70	5	120	Aluminum	7
18	47	65	5	135	Aluminum	4
14	48	70	10	135	Mild Steel	11
51	49	70	10	135	Stainless Steel	4
15	50	70	10	135	Mild Steel	10
22	51	65	10	120	Aluminum	12

Source: Authors, (2025).

III.2 APPROPRIATE MODEL SELECTION

To determine the best model based on the inserted experimental data, a statistical Fit Test was necessitated to be performed. Albeit there were many models — linear, first-order interaction (2FI), quadratic, and cubic — in the Design Expert® program, the linear model was suggested by the software after incorporating a test, namely the Sequential Model of Sum of Squares. Because the linear model was not only significant (Sequential p-value less than 0.05) but also had the least sequential p-value among all models in that test. The outcomes of the Sequential Model of the Sum of Squares test are depicted in ***Table 4***.

Table 4: Sequential Model Sum of Squares.

Source	Sum of Squares	Degree of freedom	Mean Square	F-value	p-value	Comment
Mean vs Total	3475.31	1	3475.31			
Linear vs Mean	**297.12**	**5**	**59.42**	**10.50**	**< 0.0001**	**Suggested**
2FI vs Linear	7.92	9	0.8796	0.1284	0.9986	
Quadratic vs 2FI	25.10	3	8.37	1.25	0.3087	
Cubic vs Quadratic	109.98	15	7.33	1.18	0.3630	Aliased
Residual	111.57	18	6.20			
Total	4027.00	51	78.96			

Source: Authors, (2025).

Another test as a complete part of the statistical Fit Test, is Lack of Fit. It was also conducted in this study's experimental data to cross-check whether the linear model was appropriate or not. The selected linear model had an insignificant Lack-of-Fit with a p-value of 0.184, illustrated in ***Table 5***, which was above the 0.05 threshold, indicating that the model fitted the data well without significant inconsistency. This criterion was crucial as it ensured the model's robustness and reliability in capturing the underlying data structure.

Table 5: Lack of Fit Tests.

Source	Sum of Squares	Degree of freedom	Mean Square	F-value	p-value	Comment
Linear	**208.16**	**33**	**6.31**	**1.63**	**0.1843**	**Suggested**
2FI	200.25	24	8.34	2.16	0.0831	
Quadratic	175.14	21	8.34	2.16	0.0856	
Cubic	65.17	6	10.86	2.81	0.0604	Aliased
Pure Error	46.40	12	3.87			

Source: Authors, (2025).

The performance of different regression models was also evaluated and displayed in ***Table 6,*** taking into account some metrics: standard deviation (Std. Dev.), coefficient of determination (R^2), adjusted R^2, predicted R^2, and the predicted residual error sum of squares (PRESS). These values helped in comparing the models and selecting the most appropriate one. After comparing those models, a linear model was suggested due to its balanced performance across various metrics. It had a relatively low standard deviation (2.38), indicating good precision. Moreover, the R² value of 0.5386 betokened a reasonable amount of variation explained by the model. The adjusted R² (0.4873) and predicted R² (0.4010) were both positive and higher than those of the more complex models. Besides, the Predicted R² of 0.4010 was in reasonable agreement with the Adjusted R² of 0.4873; i.e., the difference was less than 0.2, indicating better predictive power and generalizability for the minimum variety between them. Furthermore, the PRESS value (330.44) was the lowest among all models, suggesting that the linear model had the best predictive accuracy. These factors made the linear model the most suitable choice, balancing simplicity and effectiveness.

Table 6: Performance of different regression models.

Source	Standard Deviation	R²	Adjusted R²	Predicted R²	PRESS	Comment
Linear	2.38	0.5386	0.4873	0.4010	330.44	Suggested
2FI	2.62	0.5529	0.3791	-0.0175	561.33	
Quadratic	2.59	0.5984	0.3916	-0.1066	610.49	
Cubic	2.49	0.7978	0.4383	-3.3838	2418.50	Aliased

Source: Authors, (2025).

This statistical Fit Test is summarized in ***Table 7,*** where the linear model had a Sequential p-value well below the required 0.05 threshold, indicating that the model terms significantly contribute to the fit. Additionally, the model was not aliased, making it faithful. Another aforementioned criterion was an insignificant p-value — greater than 0.05 — for the Lack of Fit test, and the linear model also belonged to that condition. Furthermore, the linear model also demonstrated the close values between Adjusted R² and Predicted R², with scores of 0.4873 and 0.4010, respectively. These values indicate the best balance of fit and predictive performance by the linear model for this study's experimental data. In comparison, the first-order interaction (2FI), quadratic, and cubic models exhibited poor closeness between Adjusted R² and Predicted R² values, suggesting overfitting and impecunious predictive performance. Analyzing and interpreting the results is part of the discussions, their importance, achievements, and limitations, highlighting the innovative aspects of the practical applications of the study and the conclusions derived from them, delimiting unresolved issues. If necessary, recommendations can be proposed.

Table 7: Fit test summary.

Source	Sequential p-value	Lack of Fit p-value	Adjusted R²	Predicted R²	Comment
Linear	**< 0.0001**	**0.1843**	**0.4873**	**0.4010**	**Suggested**
2FI	0.9986	0.0831	0.3791	-0.0175	
Quadratic	0.3087	0.0856	0.3916	-0.1066	
Cubic	0.3630	0.0604	0.4383	-3.3838	**Aliased**

Source: Authors, (2025).

Apart from that statistical Fit Test, the Normal Plot of the residuals graph — depicted in ***Fig. 7*** — for the elected linear model conspicuously showed that the residuals closely follow a straight line, indicating they were approximately normally distributed. The points are symmetrically distributed around the line without significant outliers, suggesting fortuitous errors and reliable model predictions. The consistency of residuals across the range of predicted values indicates homoscedasticity, confirming that the variance of residuals was constant. This observation validated the use of the linear model, confirming its appropriateness for the data and indicating that no transformation of the response variable was necessary.

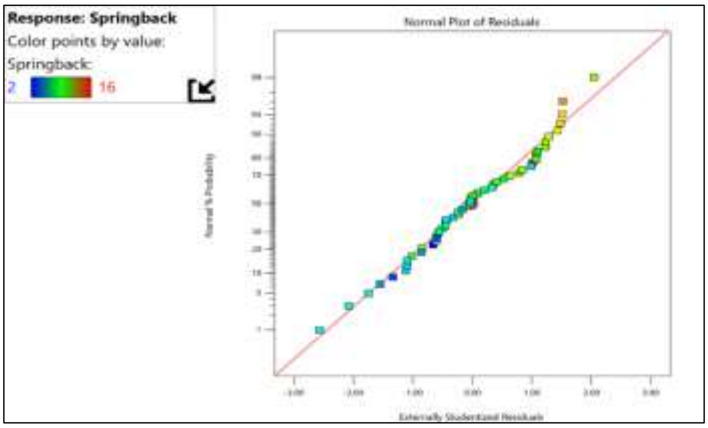

Figure 7: Statistical check for normality of residual data for the selected linear model.

Source: Authors, (2025).

Another thing is that the Residuals vs. Predicted graph — demonstrated in ***Fig. 8*** — for the selected linear model delineates that the residuals were haphazardly scattered around the horizontal axis, indicating no systematic pattern. This randomness suggested that the model's predictions were unbiased and that the linear relationship was appropriate. Additionally, the spread of the residuals is consistent across all levels of predicted values, indicating homoscedasticity, meaning the variance of the residuals remains constant. There are no noticeable outliers, — arise due to mechanical faults, human error, or instrument error [47] — confirming that the model assumptions are met, and no transformation of the response variable was needed.

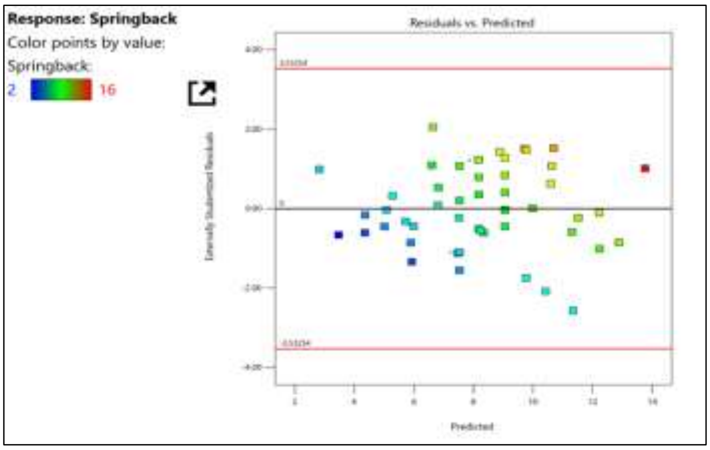

Figure 8: Scattered plot of Residuals vs Predicted data for selected linear model.
Source: Authors, (2025).

The Box-Cox graph, shown in ***Fig. 9***, for the linear model, indicates that a Lambda value of 1 is appropriate, suggesting that no transformation of the response variable was necessary. The graph reveals that the confidence interval for the optimal Lambda includes 1, confirming that the linear model without conversion was felicitous. This bolsters the assumption that the response variable assuaged the requirements for normality and homoscedasticity without any change of state. Consequently, the linear model with a Lambda value of 1 is validated as the best choice, ensuring accurate and verisimilar results.

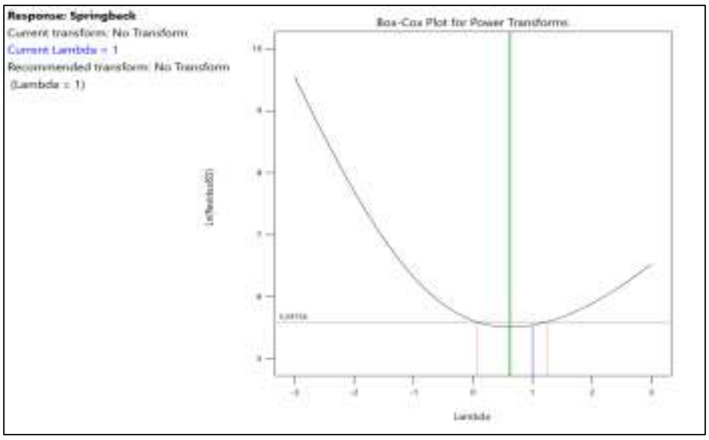

Figure 9: The Box-Cox graph for the selected linear model.
Source: Authors, (2025).

III.3 EMPIRICAL MODEL DEVELOPMENT

A relationship between the input parameters and the output parameter was developed in light of the selected linear model. Initially, the relationship as a coded equation — expressed in equation ***(4)*** — was developed by the Design Expert® program for divination springback based on several input factors.

$$\boldsymbol{Springback = 8.25 + 0.4583 \times A + 2.71 \times B - 2.00 \times C - 0.0784 \times D_1 + 0.8039 \times D_2} \quad (4)$$

In equation ***(4)***, the intercept of 8.25 provides the baseline when all factors are at their reference levels. Die gap (A) and PR/ST (B) positively influence springback with a coefficient of 0.4583 and 2.71, respectively, while set angle (C) negatively affects springback, having a coefficient of -2.00. For material types, -0.0784 signifies the effect of material being Mild Steel (coded as 1 for D_1 and 0 for D2), and 0.8039 signifies the effect of material being Aluminum (coded as 0 for D1 and 1 for D_2). When using Stainless Steel (SS), both dummy variables D_1 and D_2 were coded as -1, reflecting how springback differed compared to Mild Steel and Aluminum. It is luminous from this coded equation ***(4)*** that springback has the most sensitivity for PR/ST (B) independent factor since it has the maximum coefficient among all.

In addition to that coded equation, actual equations could also be revealed from the Design Expert® program for each material. Actual equations of Mild Steel, Aluminum, and Stainless Steel are ***(5), (6),*** and ***(7)***, respectively.

The actual equation for Mild Steel:

$$Springback_{MS} = 14.34314 + 0.091667 \times Die\ Gap + 0.541667 \times P.R/S.T - 0.133333 \times Set\ Angle \quad (5)$$

The actual equation for Aluminum:

$$Springback_{AL} = 15.22549 + 0.091667 \times Die\ Gap + 0.541667 \times P.\frac{R}{S}.T - 0.133333 \times Set\ Angle \quad (6)$$

The actual equation for Stainless Steel:

$$Springback_{SS} = 13.69608 + 0.091667 \times Die\ Gap + 0.541667 \times P.R/S.T - 0.133333 \times Set\ Angle \qquad (7)$$

III.4 EXPERIMENTAL VALIDATION OF THE DEVELOPED MODELS

In this phase of this study, experimental validation was executed for each material with the help of its corresponding actual equations. To clarify it for Mild Steel, 71.5 millimeter die gap, 8 ratios of P.R/S.T, and 135 degrees set angle were given during experimental validation; as a result, the Mild Steel specimen witnessed a 7-degree springback in the lab. The predicted springback, on the contrary, of that Mild Steel part was 7.23 degrees by using the actual springback equation ***(5)*** for the same process parameters. Therefore, the error was, technically speaking, -3.286% which was calculated by equation ***(3)***. The first row of ***Table 8*** represents this scenario. Analogously, the second and third rows depict the summary of the experimental validation settings for Aluminum and Stainless Steel. The results of the experiments also showed that the established model was credible, as the percentages of errors were extremely tiny.

Table 8: Confirmation Experiment.

No. of Experiment	**Process Parameters**					**Response factor: Springback (degree)**		
	Die gap (mm)	**P.R/ S.T**	**Set Angle (degree)**	**Materials**		**Predicted**	**Actual**	**Error (%)**
1	71.5	8	135	Mild Steel		7.23	7	-3.286
2	74.5	12	125	Aluminum		11.89	12	0.917
3	68.5	6	145	Stainless Steel		3.89	4	2.750

Source: Authors, (2025).

IV. DISCUSSIONS

The aforementioned three equations ***(5)***, ***(6)***, and ***(7)*** have numerous insights. Process-related insights are upheld below:

- From the equations, it can be inferred that if the die gap and punch radius to sheet thickness are constant, the bigger the initial set angle results the lower the springback.

$$\boldsymbol{Springback \propto -Set\ Angle}$$

where the Die gap and P.R/S.T are held constant.

- In addition to this, if the initial bending angle and the punch radius to sheet thickness are considered constant, then another relation can be drawn between springback and die gap — a higher die gap leads to a larger springback.

$$\boldsymbol{Springback \propto Die\ Gap}$$

where the Set angle and P.R/S.T are held constant.

- Further, similarly, the increment of punch radius to sheet thickness means raising the springback — if the die gap and the initial bending angle are stable. It has two implications. Firstly, if the sheet thickness is considered constant, then it can be said that the rise of the punch tip radius results in greater springback. Secondly, if the tip of the punch radius is considered constant, then springback is supposed to be decreased with respect to the increment of sheet thickness.

$$\boldsymbol{Springback \propto Tip\ of\ Punch\ Radius}$$

where the Set angle, die gap, and sheet thickness are held constant.

$$\boldsymbol{Springback \propto \frac{1}{Sheet\ thickness}}$$

where the Set angle, die gap, and the tip of the punch radius are held constant.

- These equations are also helpful in understanding the sensitivity of the process factors on springback. While P.R/S.T is the most sensitive process parameter pertinent to springback, the die gap is a less influential process factor.

V. CONCLUSIONS

This study aimed to explore springback in light of a curve-fitting equation by examining its three process parameters along with the material and assessing its trends. Our findings shed light on springback trends with process parameters. This study reveals some trends between process parameters and springback. For instance, the larger the initial set angle, the less springback there will be if the die gap and punch radius to sheet thickness remain constant. Another trend is that a higher die gap results in a larger springback, according to another relationship between springback and die gap if the initial bending angle and the punch radius to sheet thickness are taken to be constant. Moreover, in a similar vein, assuming the die gap and the initial bending angle remain constant, increasing the punch radius to sheet thickness ratio results in an increase in springback. Indirectly, it tells two other trends: first, the increase in punch tip radius leads to more springback if the thickness of the sheet is kept constant; second, in contrast, springback might be reduced in relation to the increase in sheet thickness if the punch radius tip is thought to be constant.

In addition to these process-related trends with springback, it is also found that which process parameter has supreme sensitivity and which one has inferior sensitivity on springback. While P.R/S.T is the most sensitive process parameter pertinent to springback, the die gap is a less influential process factor. The research presented in this study showcases significant advancements in the prediction of springback in air-bending operations for sheet metal, specifically focusing on Mild Steel, Aluminum, and Stainless Steel. By developing an empirical model that incorporates critical parameters such as die gap, punch radius to sheet thickness ratio (P.R/S.T), set angle, and material, this study addresses the inherent challenges associated with springback. Moreover, the developed models have practical applications that can significantly benefit the manufacturing industry. For instance, it can rectify the precision of manufacturing setups and part designs by providing authentic springback foresight, especially in scenarios where conducting tests is prohibitively expensive.

The model also serves as a valuable tool for guiding product development to achieve high precision, which is crucial for producing quality components.

V.2 LIMITATIONS OF THE STUDY

There were, howbeit, some limitations in this research. Firstly, this study was not conducted at a constant room temperature; therefore, the thermal effect could slightly distort the outcomes. Secondly, though an empirical model could be developed in this study, it was not able to foretell the absolute value of springback. This means that errors were acceptable during the validation stage. Thirdly, the Bevel Protector was kept to measure the integer angular value of all fifty-one Runs and validation. Simply put, all of the collected experimental data of springback were integers due to the Bevel Protector's measuring specification.

V.3 FUTURE WORK

Several aspects can be undertaken in future research to give more dimensions to this current study. To start with, limitations of the current study may be transcended, such as by maintaining a constant room temperature and using a highly precise Bevel Protector. Furthermore, the Finite Element Method, a sort of simulation, can be incorporated to analyze springback in forthcoming research work when numerous experimental trials are a white elephant. Moreover, a lot of experimental data can be collected to use as a training data set for developing an artificial intelligence model for predicting springback including both process parameters and mechanical properties. It will be a supercalifragilisticexpialidocious prospective research work for springback prognostication, won't it?

VI. AUTHOR'S CONTRIBUTION

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