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MODELING THE ELASTIC BEHAVIOR OF CLAY SOILS REINFORCED WITH RECYCLED RUBBER FIBERS

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ABSTRACT

This work use composite material models to forecast the modulus of elasticity of a composite made from scrap tire rubber fibers and clayey soils. The computed modulus of elasticity is juxtaposed with the reference modulus derived from experimental testing. Forecasting the effective mechanical characteristics of composites is essential in scenarios when testing is unfeasible, difficult, or expensive. The investigation encompasses many methodologies within the elasticity framework, including rheological models like Voigt, Reuss, Hirsch-Dougill, Popovics, Halpin-Tsai, Hashin, and the Bache & Napper-Christensen estimate. These models seek to forecast the effective Young's modulus of the composite system consisting of soil and rubber fibers. The highest disparities recorded are 10.66%, 12.71%, and 12.98% for both types of soil. Voigt, Hashin, and Bache calculations provide precise forecasts of the effective Young's modulus, demonstrating remarkable concordance with experimental data across various fiber volume fractions from 10% to 50%.



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I. INTRODUCTION

Used tires are abundant and concerning waste. Aggregates derived from grinding used tires are increasingly used in the field of civil engineering (geotechnical, hydraulic structures, lightweight concrete, asphalt concrete, etc.). Depending on the type of used tires, dimensions, and any separations and treatments, the physical and mechanical properties of these fibers may change. This type of material, such as fiber concrete, fiber plaster, or fiber-reinforced soils, as well as certain natural materials like bone tissues, presents a microstructure consisting of a matrix and a distribution of fibers oriented continuously in all directions (randomly) Fritsch [1]. Evaluating the effective behavior of composite using analytical methods requires a profound understanding of the various approaches existing in the abundant literature in this field. According to Gilormini et al [2], the selection of a model is influenced by various parameters, such as the geometry of the heterogeneous medium, the mechanical contrast between phases, and the volume fraction of reinforcements.

Various researchers have employed models to estimate composites, while others have created distinct models aimed at predicting the elastic properties of specific materials, such as dam concrete [3, 4], rubberized concrete [5 - 7], silica fume concrete [8], and concretes produced with different types of aggregates [9, 10]. The prediction of effective mechanical properties of composites using appropriately validated mixing laws is of great practical interest in all circumstances where tests are impossible, difficult, or expensive. Our study revolves around determining the effective Young's modulus of soil (matrix) reinforced with fibers from recycled rubber tires, comparing it to the effective Young's modulus of experimentally tested soil-fiber composites.

II. MATERIALS AND METHODS

II.1 MATERIALS USED

II.1.1 THE MATRIX PHASE (CLAY) AND ELASTIC YOUNG'S MODULUS

Two soils with distinct origins and physical characteristics were selected for analysis. The first sample was collected from the Ayaida (A) region in Oran, located in northwest Algeria. This soil has been associated with various structural issues in buildings. The second sample is Bentonite (B), provided by the Bental unit in Maghnia, also in northwest Algeria. Fig. 1 illustrates the molecular size distribution of both soils. Young's modulus (E), also known as the elastic modulus of soil, is a key parameter that reflects the stiffness of soil. It represents the ratio of stress to strain within the elastic range of the soil's behavior. This modulus is commonly used in assessing ground support systems and analyzing elastic deformations. The elastic modulus can be determined through either laboratory or in-situ testing methods, or it can be estimated using empirical correlations with other soil properties.

In laboratory settings, it is typically obtained from triaxial tests or indirectly through oedometer tests. In the field, it can be estimated using techniques such as the standard penetration test (SPT), cone penetration test (CPT), pressure meter test, or indirectly via the dilatometer test. Representative Young's modulus values for cohesive materials (in MPa), as presented by Obrzud and Truty [11], compiled from the works of Kezdi [12] and Prat et al [13]. Based on the Unified Soil Classification System (USCS), Ayaida soil is classified as low-plasticity clay (CL), with silica as its primary component. In contrast, Maghnia Bentonite is categorized as high-plasticity clay (CH). Young's modulus of Ayaida soil is 3.509 MPa, and Young's modulus of Maghnia Bentonite is 3.972 MPa.

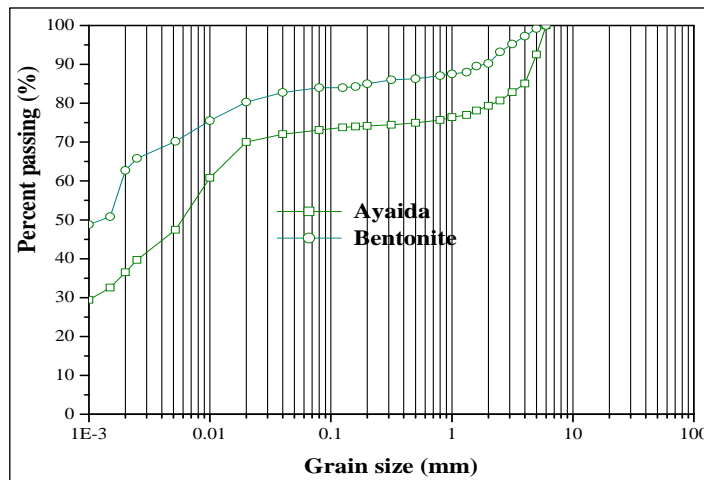


Figure 1: Grain size distributions for the studied materials.

Source: Authors, (2025).

II.1.2. THE REINFORCEMENT PHASE (RUBBER FIBERS) AND ELASTIC YOUNG'S MODULUS

The rubber fibers used in this work come from the processing of used tires by crushing. The steel fibers are separated from the powder magnetically. Fig.2 Photograph showing scrap tire rubber fiber. In addition to fine rubber particles, the powder contains textile fibers. The principle of this experimental campaign is to make test pieces of composite soils and their mixtures with different identical fiber contents (10%, 20%, 25% and 50%) of the same water content and density (homogeneous and isotropic material). The materials are packaged in hermetically sealed bags and are stored at the ambient temperature of the test room ($\sim 20^{\circ}\text{C}$). The rest time necessary to ensure homogeneous distribution of water within the sample for all composite materials .

Extensive research has been conducted on the Young's modulus of rubber tires. For example, Humphrey et al [14] examined tire chips sourced from three different suppliers by performing compressibility tests under vertical and horizontal loading and unloading conditions at low stress levels. Young's modulus values range from 1.2 to 5.1 MPa, while the average Poisson's ratio falls between 0.20 and 0.32. Triaxial tests were conducted on tire chips ranging in size from 0.08 to 2.01 inches (2 to 51 mm), as reported by Bressette [15], Ahmed [16], Masad et al [17], and Wu et al [18].

Wu et al [18] conducted compression loading tests, including loading-unloading sequences in which the confining pressure (σ_3) was gradually reduced from the initial consolidation pressure while maintaining a constant vertical load (σ_1). The initial tangent modulus derived from the stress-strain curves—comparable to Young's modulus—ranged from 0.3 to 2.5 MPa, with higher values observed under increased confining pressures. Additionally, Hernández et al [19] reported nominal mechanical properties of truck tire rubber, with Young's modulus varying from 1.97 MPa (measured at 100% strain) to 22.36 MPa. In the present study, the average Young's modulus (E_a) of waste tire rubber was found to be 2.214 MPa. The Poisson's ratio for scrap tire rubber is assumed to be 0.45.



Figure 2: Scrap tire rubber fiber.
Source: Authors, (2025).

II.2 METHODS

II.2.1 YOUNG’S MODULUS RESULTS FROM EXPERIMENTAL TESTING OF THE TWO SOILS

The test was conducted on all compacted samples following the ASTM D2435 [20] standard. The test was conducted using a conventional Oedometer with a diameter of 50 mm and a thickness of 20 mm. The load is applied in stages kept constant, successively increasing according to a defined program (0; 0.013; 0.025; 0.051; 0.102; 0.408; 0.815; 1.630) MPa. Variations in the height of the specimen are measured during the test as a function of the duration of application of the load. The principle of this experimental campaign is to make test pieces of composite soils and their mixtures with different fiber contents (0%, 10%, 20%, 25%, and 50%) identical with the same water content and density (homogeneous material and isotropic). The materials are packaged in hermetically sealed bags and are stored at the ambient temperature of the test room (~20°C). The rest time is necessary to ensure a homogeneous distribution of water within the sample for all composite materials. The composite soil is placed in a rigid envelope, a variable pressure is applied using a piston, and the subsidence observed after stabilization is measured, ASTM D 2435. The oedometric module noted E' varies according to the pressures corresponding to the stress interval (Sanglerat).

$$\Delta\sigma = \sigma_1 - \sigma \quad , \quad E' = \frac{1+e}{c_c} \frac{\Delta\sigma}{\log\left(1 + \frac{\Delta\sigma}{\sigma}\right)} \quad (1)$$

With e the vacuum index and σ the stress characterize the initial state.

$$E = E' \left(1 - \frac{2\nu^2}{\nu}\right) \quad (2)$$

Where (ν) is the Poisson's ratio of the materials. We can admit that this formula applies to soils as a first approximation. The Poisson's ratio of soils is poorly known and few determinations have been made. However, it seems that $\nu = 0.33$ constitutes a reasonable estimate. We will therefore write, adopting this value:

$$E = \frac{2}{3} E' \quad (3)$$

Table (1) compiles the Young's modulus of experimentally measured soil-fiber composite (E_c^{exp}). These first tests make it possible to trace the reference curves to then compare them with tests on soil – rubber fiber composites with different volume fractions. Bibliographic data for different experimentally tested soils are compiled in Table 2. The mechanical data are E_m the Young's modulus of the matrix, E_a that of the rubber fibers and E_c^{exp} the Young's modulus of the soil measured experimentally. As well as different volume fractions for soil-fiber composites. In our work we tested two types of clay soils with Young's modulus (3.509 and 3.972) MPa. Each type of matrix soil is mixed with reinforcement of rubber fibers having Young's modulus (2.214) MPa. Which corresponds to a contrast ratio between the two phases varying from (E_a/E_m) varying from 0.56 to 0.63.

Table 1: the modulus of elasticity of soil-fiber composite measured experimentally.

Ref	σ (MPa)	$\Delta\sigma$ (MPa)	e	c_c	E_c^{exp} (MPa)
A1	1.63	0.518	0.595	3.99	3.509
A2			0.285	6.64	3.185
A3			0.550	6.98	3.295
A4			0.689	12.62	3.221
A5			0.357	7.31	3.203
B1			0.799	2.66	3.972
B2			0.773	5.65	3.472
B3			0.698	5.32	3.455
B4			0.853	7.64	3.396

Source: Authors, (2025).

Table 2: Experimental characteristics and volume fraction for soil-fiber composites.

Ref.	V_a	V_m	E_a (MPa)	E_m (MPa)	E_c^{exp} (MPa)
A ₁	0	1	2.214	3.509	3.509
A ₂	0.1	0.9			3.185
A ₃	0.2	0.8			3.295
A ₄	0.25	0.75			3.221
A ₅	0.5	0.5			3.203
B ₁	0	1		3.972	3.972
B ₂	0.1	0.9			3.472
B ₃	0.2	0.8			3.455
B ₄	0.25	0.75			3.396
B ₅	0.5	0.5			3.316

Source: Authors, (2025).

II.2.2 CHOICE OF ANALYTICAL MODELS

As noted by Gilormini and Bréchet [2], model selection is influenced by several factors, notably the geometry of the heterogeneous medium, the mechanical contrast between the matrix phases (soils), and the volume fraction of the reinforcements (rubber fibers). Remember that in our case, we are studying a heterogeneous composite material (soil-rubber fibers from worn tires) composed of two homogeneous phases: the fibers (or the inclusion; phase a) of volume fraction (V_a), embedded in the soil (or matrix, phase m). Each of these two phases presents a linear, homogeneous and isotropic elastic behavior. The two phases are assumed to be perfectly stuck together (matrix/inclusion bond, perfect adhesion is assumed), which justifies the biphasic representation. Used tire rubber fibers are evenly distributed in the soil matrix.

The multi-phase description of composite materials made up of an elastic matrix. The determination of the effective properties then consists in this case of defining the homogeneous equivalent soil-fiber behavior based on the characteristics of the soil and the rubber fibers of the worn tires. The two mixing laws of Voigt and Reuss are simple and incapable of determining with acceptable precision the modulus of elasticity of the composite because they do not take into account the morphology of the material, the discontinuity of the reinforcement, nor its orientation and of the true nature of matrix/reinforcement interface, these two terminals are enriched by combining them with each other. Two combinations proposed by Larrard [21]. The effective elastic modulus for both models (Voigt and Reuss) is given by equations (4) and (5). The biphasic models of Hirsch-Dougill, Popovics , Halpin-Tsai, Hashin & Bache and Napper-Christensen originally designed for particle composites Nielsen et al [22], propose effective elastic modulus of the composite by equations (6), (7), (8), (9) and (10) successively.

$$E_{C\text{Voigt}} = E_m V_m + E_a V_a \tag{4}$$

$$E_{C\text{Reuss}} = \frac{E_m}{V_m} + \frac{E_a}{V_a} \tag{5}$$

$$E_{C\text{Hirsh-Dougill}} = \frac{1}{2} \left(\frac{1}{E_{C\text{Voigt}}} + \frac{1}{E_{C\text{Reuss}}} \right) \tag{6}$$

$$E_{C\text{Popovics}} = \frac{1}{2} (E_{C\text{Voigt}} + E_{C\text{Reuss}}) \tag{7}$$

$$E_{C\text{Halpin-Tsai}} = \frac{3}{8} E_{C\text{Voigt}} + \frac{5}{8} E_{C\text{Reuss}} \tag{8}$$

$$E_{CHalpin} = \frac{(E_a + E_m) + (E_a - E_m)V_a}{(E_a + E_m) - (E_a - E_m)V_a} E_m \quad (9)$$

$$E_{CHalpin} = E_m^{V_m} E_a^{V_a} \quad (10)$$

Where E_m represents the modulus of elasticity of the soil matrix, E_a is the modulus of elasticity of the fiber phase, E_C is the modulus of elasticity of soil-fibers, V_m is the volume fraction of the soil as matrix and V_a is the volume fraction of the rubber fibers of the reinforcement phase. E_C Takes the indices of the composite model used for estimation of the elastic modulus. For example, E_C Voigt expresses the elastic modulus of soil-fibers estimated using the Voigt composite model.

III. RESULTS OF PREDICTIVE APPROACHES AND DISCUSSION

III.1 IMPACT OF FIBER CONTENT ON THE DRY DENSITY OF TWO CLAY SOILS

Based on the results depicted in Fig. 3, it is evident that the dry density of fiber-reinforced soils gradually decreases with the increasing content of fibers for both soil types A and B. This reduction is attributed to the decrease in the average unit weight of solids within the mixture. The test results indicate that the inclusion of fibers leads to a decrease in the maximum dry density in both soils A and B, primarily due to the lower density of the added fibers. The decline in the mixture's density can be attributed to several factors. Firstly, the rubber fibers have a smaller specific gravity compared to the soil. In this study, the dry density of clay B and A, and rubber fibers were 1.98 g/m³, 1.93 g/m³, and 1.20 g/cm³, respectively. Secondly, the flexibility of rubber fibers contributes to enhanced compaction efficiency, resulting in a reduced maximum dry density of the soil and fiber mixture.

Thirdly, the lower optimum moisture content of clay and fibers, influenced by the poor water absorption of rubber fibers compared to clay. For instance, an increase in the content of tire rubber fibers from 10% to 50% leads to a reduction in density by 26.67% and 26.88% for soil A and B, respectively. These findings align with previous studies [23, 24] indicating that, while the optimum water content remains unchanged, the maximum dry unit weight of the mixture decreases by about 0.3 g/cm³ with the addition of 20% rubber by weight . Bekhiti et al [25] also observed a decrease in dry density with an increase in waste tire rubber fiber content. The highest dry density value, 1.29 g/cm³, was achieved with a 30% PVC waste aggregate content. Interestingly, this maximum dry density is 17.83% lower than the unreinforced samples [26].

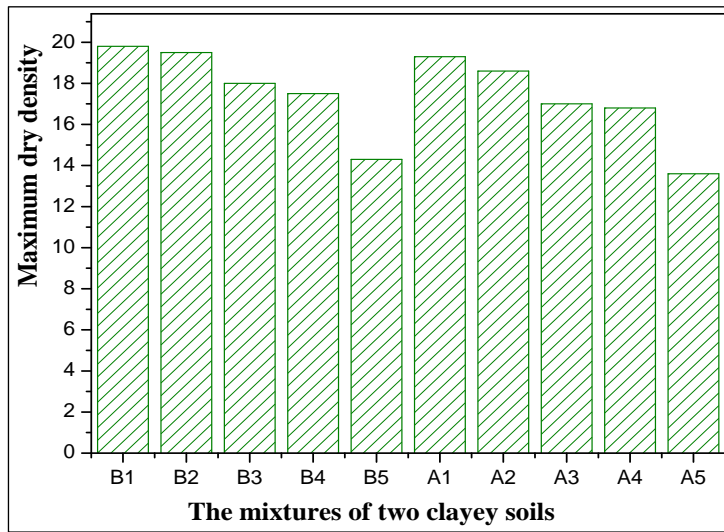


Figure 3: Variation of dry density of different mixtures. Source: Authors, (2025).

III.2 COMPARISON OF ANALYTICAL MODELS WITH EXPERIMENTAL RESULTS.

Table 3 shows our results from the application of analytical approaches and those obtained by experimental method. To identify the most accurate model for calculating the effective Young's modulus, the differences between the predicted values from various models and the experimental results were computed (see Table 4). The average relative differences between the measured experimental E_c^{Exp} Young's modulus and the numerical predictions E_c^{Cal} are calculated by the relation below:

$$E_{Cart} = \left| \frac{E_C^{Cal} - E_C^{Exp}}{E_C^{Exp}} \right| \times 100 \quad (11)$$

Figs. 4 and 5 give the evolution of the effective Young's modulus obtained by the different predictive approaches processed and the experimental results as a function of the volume fraction of the rubber fibers (reinforcement) (V_a) which takes the values: 0-0.1-0.2-0.25 and 0.50 for a Young's modulus is 2.214 MPa.

If we examine the results presented in Table 4 we notice that the maximum difference is reached with the combined models of Reuss and Hirsch is worth 93.66% and 88.08% respectively, this is a great value. The two models are not capable of predicting the effective behavior of the composites (soil-fiber). As for the Popovics and Halpin-Tsai models, they gave percentages close to half, for example, with 10% fibers, this gave a difference of 42.21% and 54.29% for soil A, as for the ground B; this gave 46.83% and 54.58% respectively. The Voigt, Hashin, and Bache approximations demonstrate a minimal deviation range, spanning from 0.00% to 10.66% for Voigt, 0.00% to 12.717% for Hashin, and 0.00% to 12.988% for Bache, considering two types of soils, A and B. These three models prove effective in predicting the composite materials (soil-rubber fibers) effective Young's modulus.

Table 3: Effective modulus of elasticity (MPa): Comparative analysis of various analytical models against experimental results.

Ref.	$E_{C\text{Voigt}}$	$E_{C\text{Reuss}}$	$E_{C\text{Hirsch}}$	$E_{C\text{Popovics}}$	$E_{C\text{Halpin-Tsai}}$	$E_{C\text{Hashin}}$	$E_{C\text{Bache}}$	E_c^{exp}
A ₁	3.510	0.281	0.526	1.898	1.495	3.600	3.601	3.601
A ₂	3.380	0.301	0.544	1.840	1.455	3.353	3.352	3.386
A ₃	3.250	0.320	0.560	1.783	1.419	3.204	3.200	3.194
A ₄	3.190	0.331	0.568	1.757	1.400	3.132	3.128	3.222
A ₅	2.860	0.370	0.618	1.614	1.301	2.797	2.788	3.203
B ₁	3.971	0.251	0.474	2.111	1.646	3.973	3.973	3.971
B ₂	3.801	0.271	0.491	2.035	1.594	3.751	3.748	3.471
B ₃	3.620	0.291	0.510	1.957	1.540	3.544	3.535	3.454
B ₄	3.530	0.301	0.521	1.916	1.511	3.446	3.433	3.397
B ₅	3.101	0.351	0.580	1.723	1.380	2.983	2.966	3.315

Source: Authors, (2025).

Table 4: Percentage deviations in Young's modulus between analytical model predictions and experimental results.

Ref.	$E_{C\text{Voigt}}$	$E_{C\text{Reuss}}$	$E_{C\text{Hirsch}}$	$E_{C\text{Popovics}}$	$E_{C\text{Halpin-Tsai}}$	$E_{C\text{Hashin}}$	$E_{C\text{Bache}}$
A ₁	0.000	91.880	84.980	45.940	57.420	0.000	0.000
A ₂	6.110	90.530	82.950	42.210	54.290	5.300	5.120
A ₃	1.370	90.340	83.000	45.850	56.970	2.730	2.880
A ₄	1.110	89.860	82.330	45.480	56.580	2.720	2.920
A ₅	10.660	88.500	80.670	49.580	59.110	12.720	12.990
B ₁	0.000	93.660	88.080	46.830	58.540	0.000	0.000
B ₂	9.340	96.170	85.830	41.420	54.110	8.080	7.920
B ₃	4.790	91.560	85.210	43.380	55.430	2.600	2.290
B ₄	4.020	91.110	84.630	43.550	55.440	1.440	1.060
B ₅	6.720	89.390	82.480	48.060	57.420	0.000	10.580

Source: Authors, (2025).

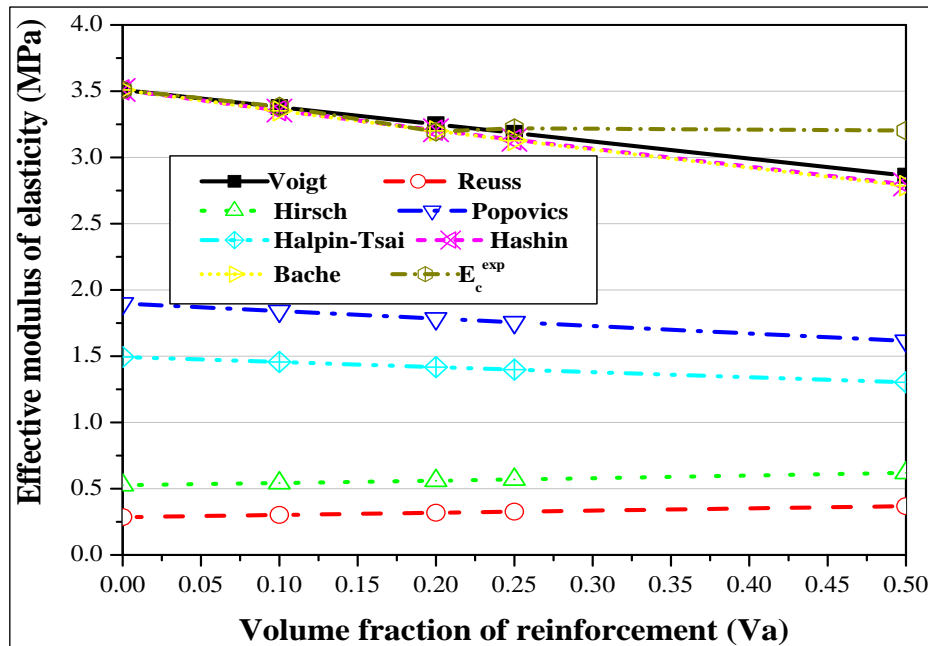


Figure 4: Comparison of analytical and experimental results for effective elastic moduli as a function of the volume fraction of the reinforcement in Ayaida clay.

Source: Authors, (2025).

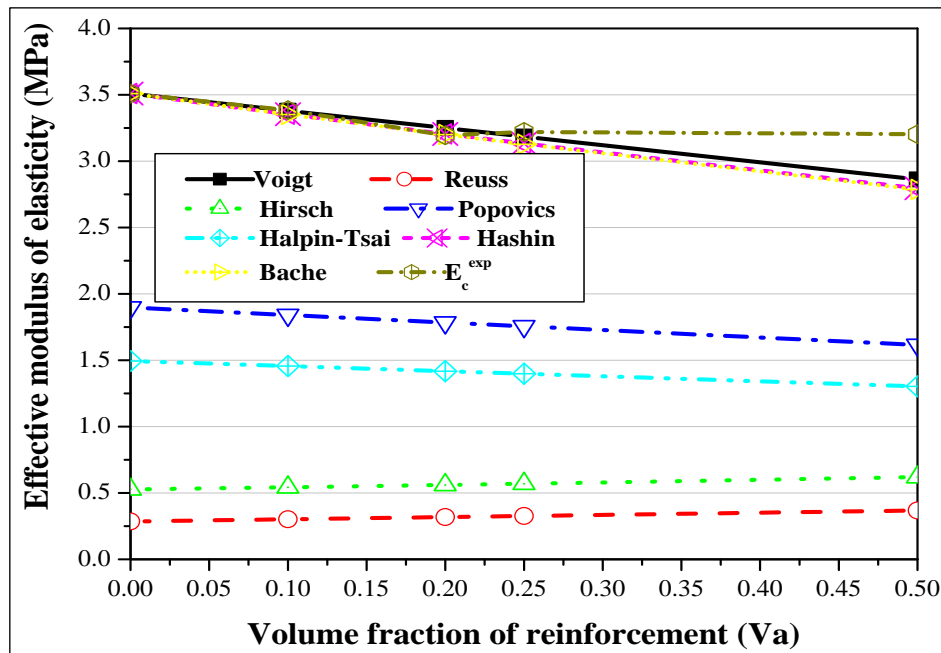


Figure 5: Comparison of analytical and experimental results for effective elastic moduli as a function of the volume fraction of the reinforcement in Bentonite of Meghnia.

Source: Authors, (2025).

IV. CONCLUSIONS

Our comparative study allows us to conclude that:

- The increase in fiber content results in a reduction of the dry density in both varieties of clay soil, mostly due to the reduced specific gravity and unit weight of the fibers.
- The elevation of rubber fiber content results in a reduction of the modulus of elasticity in soils A and B.
- The results obtained from predictive approaches for calculating the modulus of elasticity in two clayey soils reinforced with rubber fibers from worn tires indicate that the Voigt, Hashin, and Bache models are in closer agreement with experimental results, showing a maximum deviation of 10.66% for Voigt, 12.717% for Hashin, and 12.988% for Bache in the case of soil A. In soil B, the maximum differences are 6.72%, 0%, and 10.58% with the same models when using 50% fibers. The Voigt and Hashin models exhibit superior accuracy, particularly for clayey soils incorporating fibers.

This study involves predicting the effective mechanical properties of composites using well-validated mixing laws and is of significant practical interest in situations where testing is impossible, difficult, or expensive. In this study, some of these composite models provide accurate calculations and closer values for the moduli of elasticity. They can also calculate the modulus of elasticity for clayey soil reinforced with waste rubber fibers, providing a feasible alternative to prior models with requisite assumptions. Future studies could investigate some of the composite models not examined in this paper for clay soil reinforced with rubber fibers to forecast the modulus of elasticity of elasticity and compare the results with experimental data. Future studies could develop composite models specifically tailored for soils, rather than relying on models primarily designed for concrete, to precisely forecast the modulus of elasticity for clay composites augmented with rubber fibers derived from recycled tires.

V. AUTHOR'S CONTRIBUTION

Conceptualization: Melik Bekhiti, Abdelhalim Bensaada, and Ali Smaida

Methodology: Author One and Author Two.

Investigation: Author One and Author Two and Author Three.

Discussion of results: Author Two and Author Three.

Writing – Original Draft: Author One and Author Two.

Writing – Review and Editing: Author One and Author Two.

Resources: Author Two and Author Three.

Supervision: Author Two and Author Three.

Approval of the final text: Author One, Author two and Author Three.

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