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RESEARCH ARTICLE OPEN ACCESS

STRUCTURAL OPTIMIZATION USING GREY WOLF OPTIMIZER AND ITS VARIANTS

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ABSTRACT

Now a days, many metaheuristic optimization algorithms are invariably used in the field of engineering and data science. In structural mechanics, many stochastic optimization algorithms are employed for solving the complex numerical problems. Suitable optimization algorithms must be selected as the structural mechanics problems are often having non-linear objective function, constraints are having complex behavior and many design variables are involved in it. In present study, the capability and competency of standard grey wolf optimizer (GWO) with modified grey wolf optimizer (mGWO) and grey wolf optimizer with variable weight (vwGWO) is observed through several benchmarked numerical case studies. To control the geometric constraints, penalty method is employed in the study. The performance of the standard GWO algorithm is compared with the results obtained by the modified grey wolf optimizer and grey wolf optimizer with variable weight.

I. INTRODUCTION

Since last three decades, metaheuristic optimization techniques are evolved very fast with prominent features. Some of them are very popular techniques like, Genetic Algorithm (GA) [1], Particle Swarm Optimization (PSO) [2], Artificial Bee Colony (ABC) [3], Ant Colony Optimization (ACO) [4], Grey Wolf Optimizer (GWO) [5] etc., not only in the field in computer science but also in the field of engineering, technology and structure mechanics as well. The metaheuristic optimization algorithms are very simple to implement and also flexible.

They are mostly inspired by simple concepts and this allows the researchers to simulate the different concepts from nature, to propose newer metaheuristic approach, to hybridize two or more metaheuristic algorithms or to improve the existing metaheuristic optimization algorithms [5]. Many researches were also performed in the field of optimization considering various case studies and their results are compared [6]. In article, Grey wolf optimizer explained briefly in section II followed by its variants, mGWO and vwGWO in sections III and IV respectively. Three case studies are discussed implementing GWO, mGWO and vwGWO in section V. The competency of the GWO, metaheuristic algorithm with mGWO and vwGWO is discussed in section VI.

II. GREY WOLF OPTIMIZER

Grey wolf optimizer is proposed by [5] which is based on the social and hunting behavioral of grey wolves. Grey wolves are apex predators belongs to Canidae family and hence they are at the top of the food chain. Grey wolves are living in groups. The group size on an average of the wolves is generally 5 to 12.

Grey wolf optimizer imitates the social behavior and hunting mechanism of the grey wolves. The optimum value search mechanism of the algorithm is based on the three steps of hunting technique adopted by the grey wolves:

- Tracking, chasing and approaching the prey.
- Pursuing, encircling and harassing the prey until it stops the motion.
- Attacking the prey.

The mathematical model of the grey wolf optimizer is outlined as follows:

II.1 SOCIAL HIERARCHY

The optimum solution or the fittest solution of the problem is decided as per the social hierarchy of the wolves in a pack to model the optimization algorithm mathematically. In a pack of wolves, the leader wolf is considered as α wolf. The position of the α wolf is noted as fittest solution from the possible solutions. The second-best solution is considered as the position of β wolves which are the subordinate wolves to leader wolf and help in decision making and other group activities. The lowest ranking of the wolves in a pack is known as ω. These are the wolves who play the role of scapegoat or sometimes work as baby sitters. The third best solution is considered as the position of δ wolves. If the wolf is not α wolf, β wolf or ω wolf, he/she is considered as δ wolf. This type of wolves scouts the pack and looking the boundary of the territory. In case of any danger, they warn the pack.

II.2 ENCIRCLING THE PREY

During hunting, the wolves encircle the prey. The encircling behavior of the wolves is modelled mathematically as follows:

$$\vec{D} = \left|\vec{C}.\overrightarrow{X_p}(i) - \vec{X}(i)\right| \tag{1}$$

$$\vec{X}(i+1) = \overrightarrow{X_p}(i) - \vec{A}.\vec{D} \tag{2}$$

$$\vec{X}(t) = \frac{\overrightarrow{X_1} + \overrightarrow{X_2} + \overrightarrow{X_3}}{3} \tag{3}$$

Where, i indicates the current iteration. $\vec{A}$ and $\vec{C}$ are the vectors coefficients. $\overrightarrow{X_p}$ indicates the position of the prey and $\vec{X}$ is the position vector of the grey wolf. The vectors coefficients $\vec{A}$ and $\vec{C}$ are calculated as per following expressions:

$$\vec{A} = 2\vec{a}\,.\vec{r}_1 - \vec{a} \tag{4}$$

$$\vec{C} = 2\,.\vec{r}_2 \tag{5}$$

Here, $\vec{r}_1$ and $\vec{r}_2$ are arbitrary vectors in the domain of [0,1] and $\vec{a}$ is the decrement vector which varies linearly from 2 to 0.]

II.3 HUNTING TECHNIQUE

In hunting of the prey, the hunt is generally guided by the α wolf. β wolves and δ wolves also participate in hunting. Initially, there is no idea about the optimum(prey) position in a search space and hence to model the mechanism mathematically, the initialization of the hunt by the wolves in the search space are recorded as initial solution and the positions assigned as per the fitness of the solution. The best initial solution is recorded as α-position, second best solution is recorded as β-position and third best solution is recorded as δ-position.

General steps of GWO algorithm as follows:

Grey wolf pack formation
Initialize the position of wolves
Fitness calculation
Position record
Repeat
  Update the random parameters
  Fitness calculation
  Position record appraisal
    α-wolves (the best solution),
    β-wolves, (the second best solution)
    δ-wolves (the third best solution)

until
Convergence criterion achieved

The controlling parameter ‘a’ reduces linearly from 2 to 0 when the iterations are being carried out.

$$a = 2\left(1 - \frac{i}{N}\right) \tag{6}$$

Where, i is the $iteration$ and N is $maximum\ number\ of\ iterations$.

III. MODIFIED GREY WOLF OPTIMIZER (mGWO)

Modified grey wolf algorithm is proposed by [7] mainly focused on the good balance between proper exploration and exploitation to find the global optima of real problems. In standard GWO algorithm, the coefficient vector $\vec{A}$ randomly varies between '-a' to 'a' and simultaneously over the course of iterations, $\vec{a}$ varies linearly from 2 to 0. Balancing between $\vec{A}$ and $\vec{a}$ needs much attention and fine adjustment is required to avoid the solution falls under local optima. To overcome this issue, the researchers suggested to use exponential decay function for tuning the parameter 'a'. Earlier, In standard GWO, the value of 'a' decreases from 2 to 0 linearly as per equation (6). as per the suggested exponential decay function (Mittal N. et al 2016) the expression is as follows:

$$a = 2\left(1 - \frac{i^2}{N^2}\right) \tag{7}$$

Where, i is the $iteration$ and N is $maximum\ number\ of\ iterations$. The suggested approach is adopted in the proposed research and applied to the benchmarked problems in section V to derive the competency and worthiness of mGWO.

IV. GREY WOLF OPTIMIZER WITH VARIABLE WEIGHT (vwGWO)

To make the GWO algorithm very efficient to handle the variety of problems with multi variables and high complexity of the real-life problems of engineering, a new hypothesis related to search mechanism of the grey wolf optimizer by Gao Z. M. et al in 2019 [8] is proposed as improved grey wolf optimization with variable weight (vwGWO). In standard GWO algorithm as per equation (2.3), dominants play an equal role in search mechanism of grey wolf optimizer. During exploration process, in the initial phase of algorithm, the alpha wolf (α) has the better position than β wolf or δ wolf in each iteration cycle. In searching process α wolf is the better contributor so his/her position should be given weight more than the positions of β wolf and δ wolf in next search cycle [8].

$$\vec{X}(t+1) = w_1\overrightarrow{X_1} + w_2\overrightarrow{X_2} + w_3\overrightarrow{X_3} \tag{8}$$

$$w_1 + w_2 + w_3 = 1 \tag{9}$$

The weights w_1, w_2 and w_3 are associated with α wolf, β wolf and δ wolf respectively and must satisfy the condition, $w_1 \geq w_2 \geq w_3$. For vwGWO, the proposed equations for variable weights are as follows:

$$\begin{aligned} w_1 &= \cos\theta, \\ w_2 &= \frac{1}{2}\sin\theta\cos\varphi, \\ w_3 &= 1 - w_1 - w_2 \end{aligned} \tag{10}$$

Where,

$$\theta = \frac{2}{\pi}\arccos\frac{1}{3}\cdot\arctan(i)\,; \tag{11}$$

$$\varphi = \frac{1}{2}\arctan(i) \tag{12}$$

V. NUMERICAL CASE STUDIES

Three cases are discussed to establish the competence of the GWO with its variants i.e. mGWO and vwGWO. MATLAB code is prepared for each numerical case and 10 runs are taken for varying search agents and number of iterations.

Case: I CANTILEVER BEAM

Problem statement:

Figure 1 shows stepped hollow cantilever beam with constant thickness, of square cross section. It is subjected to a vertical load (P) at free end of the beam structure. The problem is of weight minimization and design variables are height of different beam sections. The thickness of the beam elements is fixed as t = 2/3.

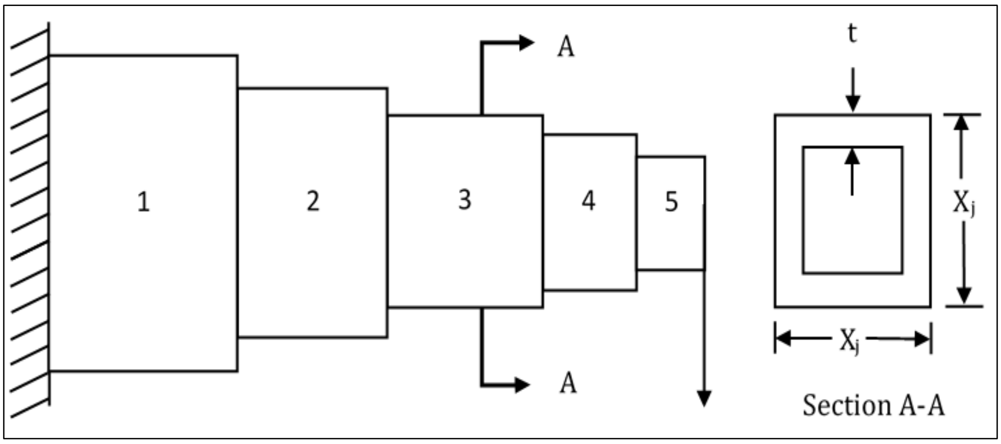

Figure 1: Cantilever Beam.
Source: Authors, (2026).

Objective Function: Minimization

$$f(x) = f(x_1, x_2, x_3, x_4, x_5) = 0.0624\,(x_1 + x_2 + x_3 + x_4 + x_5)$$

Constraints:

$$C = \frac{61}{x_1{}^3} + \frac{37}{x_2{}^3} + \frac{19}{x_3{}^3} + \frac{7}{x_4{}^3} + \frac{1}{x_5{}^3} - 1 \le 0$$

Range:

$$0.01 \le x_j \le 100$$

Table 1: Comparison of the results for cantilever beam problem.

	GWO	**mGWO**	**vwGWO**
X_1 (Element – 1)	6.0465	5.9381	6.0199
X_2 (Element – 2)	5.2891	5.3576	5.2858
X_3 (Element – 3)	4.4449	4.5531	4.5513
X_4 (Element – 4)	3.5429	3.4594	3.5022
X_5 (Element – 5)	2.1544	2.1772	2.1177
$F_{min.}$	1.3402	1.3407	1.3402

Source: Authors, (2026).

It is noticed that to optimize the weight of stepped cantilever beam, GWO requires 25 wolves and 50 iterations to reach the optimum function value. mGWO requires 25 wolves and 100 iterations to reach the optima and vwGWO needs 25 wolves and 100 iterations to reach the optimum solution.

Case: II TUBULAR COLUMN DESIGN

Problem statement:

Figure 2 shows tubular column section which carries a vertical load P. The column is joined by means of hinged joints at both the ends. The mean dia. of the column is kept between 2 cm to 14 cm and the thickness ranged between 0.2 cm to 0.8 cm. The induced stress in the tubular column should not be more than the buckling stress and the yield stress. The total cost of the column considering both, material cost and construction cost, can be taken as 5W + 2d, where W stands for weight in kgf and d is mean dia. of the column in cm. The cost of the component is taken as objective of the problem which should be minimize. The material property and related particulars of the column is given below:

P = 2500 kgf (compressive);
l = 250 cm;
Yield stress = 500 kgf/cm^2;
Modulus of Elasticity= 0.85 X 106 kgf/cm^2;
Weight density = 0.0025 kgf/cm^3;

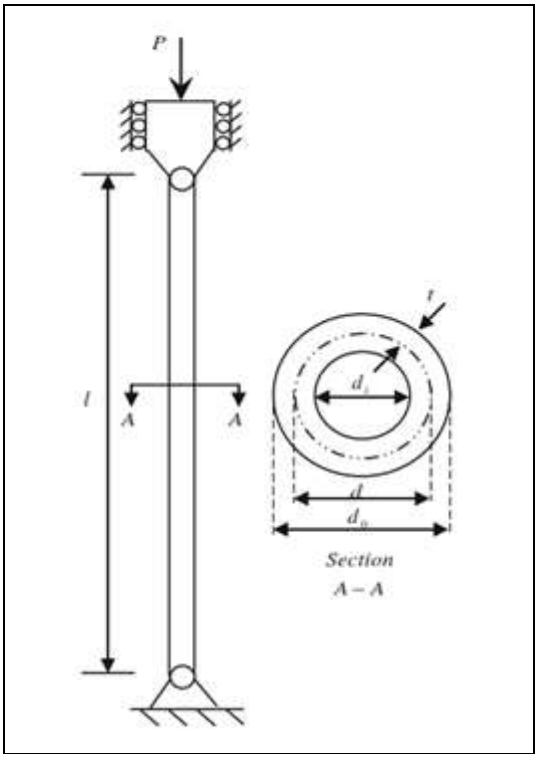

Figure 2: Tubular column design.
Source: Authors, (2026).

Objective Function: Minimization

$$f(x) = f(x_1, x_2) = 9.82\, x_1 x_2 + 2x_1$$

Where,

$$X = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} d \\ t \end{Bmatrix}$$

Constraints:

$$c_1 = \frac{2500}{\pi x_1 x_2} - 500 \leq 0$$

$$c_2 = \frac{2500}{\pi x_1 x_2} - \frac{\pi^2 (0.85 \times 10^6)(x_1{}^2 + x_2{}^2)}{8(250)^2} \leq 0$$

$$c_3 = \frac{2}{x_1} - 1 \leq 0$$

$$c_4 = \frac{x_1}{14} - 1 \leq 0$$

$$c_5 = \frac{0.2}{x_2} - 1 \leq 0$$

$$c_6 = \frac{x_2}{0.8} - 1 \leq 0$$

Range:

$$2 \leq x_1 \leq 14$$

$$0.2 \leq x_2 \leq 0.8$$

Table 2: Comparison of the results for tubular column problem.

	GWO	**mGWO**	**vwGWO**
d (mean diameter)	5.4511	5.4501	5.4495
t (thickness)	0.29204	0.29222	0.29229
$F_{min.}$	26.5351	26.54	26.5404

Source: Authors, (2026).

It is noted that to optimize tubular column design, standard GWO requires 25 wolves and 200 iterations to reach the optimum function value. mGWO requires 25 wolves and 100 iterations to reach the optima and vwGWO needs 25 wolves and 200 iterations to reach the optimum solution.

Case: III THREE BAR TRUSS

Problem statement:

Figure 3 presents three bar truss. The objective is to find the optimum cross sectional area, to minimize the volume of the three bar truss structure. Each member length of the structure is 100 cm. The structure is subjected to load (F) of 2 kN/cm^2 and the permissible stress (σ) is not exceed to 2 kN/cm^2.

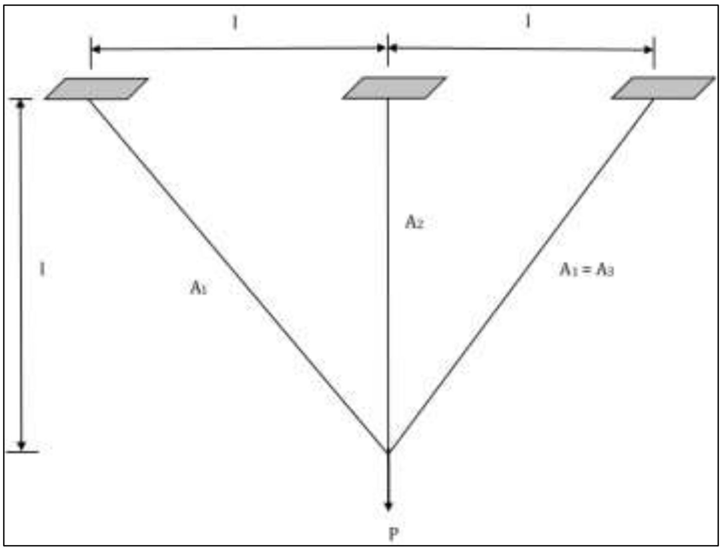

Figure 3: Three bar truss.
Source: Authors, (2026).

Objective Function: Minimization

$$f(x) = f(x_1, x_2) = \left(2\sqrt{2}x_1 + x_2\right) \times l$$

Where,

$$X = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix}$$

Constraints:

$$c_1 = \frac{\sqrt{2}x_1 + x_2}{\sqrt{2}{x_1}^2 + 2x_1x_2} F - \sigma \leq 0$$

$$c_2 = \frac{x_2}{\sqrt{2}{x_1}^2 + 2x_1x_2} F - \sigma \leq 0$$

$$c_3 = \frac{1}{x_1 + \sqrt{2}x_2} F - \sigma \leq 0$$

Range:

$$0 \leq x_1, x_2 \leq 1$$

Table 3: Comparison of the results for three bar truss structural problem.

	GWO	**mGWO**	**vwGWO**
A_1 (C/S area of member 1 & 3)	0.78776	0.78453	0.78947
A_2 (C/S area of member 2)	0.41085	0.42026	0.40601
$F_{min.}$	263.897	263.925	263.8969

Source: Authors, (2026).

It is noted that to optimize the cross-sectional area of the members of three bar truss structure, GWO requires 25 wolves and 100 iterations to reach the optimum function value. mGWO requires 20 wolves and 100 iterations to reach the optima and vwGWO needs 25 wolves and 100 iterations to reach the optimum solution.

VI. CONCLUSION

The present work explores the capability of metaheuristic optimization algorithms, GWO and also employed its modified versions known as modified GWO (mGWO) and improved GWO with variable weight (vwGWO). In case of stepped cantilever beam, GWO is found competent enough against mGWO and vwGWO whereas for tubular column design and three bar truss, mGWO is found competent in terms of less search agents and minimum iterations required to converge the solution. However, the effect of the upgraded versions i.e. mGWO and vwGWO is effective in case of multi-objective multiple constraints wherein the various variables affect the domain differently in the domain simultaneously.

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VIII. AUTHOR CONTRIBUTIONS

Conceptualization: Bhavik D. Upadhyay, Sonal T. Dave, Vishal J. Pandya, Jignesh G. Parmar, Hasmukh P. Koringa and Miral J. Patel.
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Investigation: Bhavik D. Upadhyay and Sonal T. Dave.
Data collection and Analysis: Bhavik D. Upadhyay and Sonal T. Dave.
Discussion of results: Bhavik D. Upadhyay, Miral J. Patel and Sonal T. Dave.
Manuscript Writing – Original Draft: Bhavik D. Upadhyay and Sonal T. Dave.
Writing – Review and Editing: Bhavik D. Upadhyay, Hasmukh P. Koringa and Vishal J. Pandya.
Overall formatting: Vishal J. Pandya, Hasmukh P. Koringa and Jignesh G. Parmar.
Resources: Vishal J. Pandya, Miral J. Patel and Jignesh G. Parmar.

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