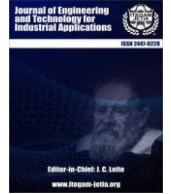




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A NEUTROSOPHIC GENERALIZED GAMMA APPROACH TO MODELING PRL, LH, AND TESTOSTERONE IN BEAGLES

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ABSTRACT

The Classical Generalized Gamma Distribution (GGD) has been widely applied in survival analysis, hydrology, and reliability engineering due to its flexibility in handling diverse data patterns. To incorporate uncertainty and imprecision commonly encountered in practical scenarios, this study introduces an extended form of the distribution, termed the Neutrosophic Generalized Gamma Distribution (NGGD). The NGGD is employed to investigate the secretion behavior of prolactin (PRL), luteinizing hormone (LH), and testosterone in male dogs. Analysis reveals that PRL exhibits a non-pulsatile, constitutive release pattern, which contrasts with the mixed pulsatile secretion observed in humans. The NGGD-based probability density functions further enable more accurate differentiation between control and drug-treated groups, aligning with established physiological evidence. These findings highlight the effectiveness of the NGGD in capturing biological variability under uncertainty, with potential applications extending to risk assessment, quality monitoring, and supply chain analysis.



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I. INTRODUCTION

The study of lifetime data and reliability analysis has become more popular across various fields, including engineering, medicine and economics to survive in the competitive market for an extended period. Accurate modeling of the time until an event occurs, such as component failure or system breakdown, is essential for optimizing performance, improving safety, and reducing costs. Over the past century, several key probability distributions have been developed to meet the diverse modelling needed in these applied fields. Classical distributions such as the Exponential, Gamma, and Weibull distributions have been widely used to model such phenomena. The GGD [1] extends the classical Gamma distribution by introducing additional shape parameters that allow for greater flexibility in modeling various hazard rate behaviors, including increasing, constant, and decreasing risks. This flexibility makes the GGD particularly suited for applications where data exhibit non-standard patterns that simpler distributions cannot adequately address.

As systems become more complex and their failure mechanisms more intricate, the ability to model lifetimes with a distribution that adapts to these complexities is invaluable. Traditional statistical distributions often struggle to represent realworld phenomena characterized by incomplete or ambiguous information accurately. This gap necessitates the development of more robust models that can effectively capture the complexities of uncertain data. One such advancement is the application of neutrosophic logic, which extends classical and fuzzy logic by incorporating degrees of truth, falsity, and indeterminacy, making it ideal for situations involving incomplete or uncertain information. Neutrosophic Statistics [2], based on neutrosophic logic, enable data to be modeled within intervals that reflect these uncertainties. This innovative philosophical domain extends the principles of fuzzy and intuitionistic fuzzy logic [3-7]. Smarandache [8-11] further developed the core concepts of neutrosophic sets across various fields.

In recent years, there has been a rapid growth in the development of neutrosophic distributions. These distributions are designed to model different types of phenomena where there is uncertainty or incomplete information. Many researchers have introduced important neutrosophic probability distributions to better analyze these types of data, providing more flexibility and accuracy compared to traditional methods. Some examples are neutrosophic beta [12], neutrosophic Rayleigh distribution [13], neutrosophic Exponential distribution [14], neutrosophic Kumaraswamy distribution [15], and neutrosophic Weibull distribution [16]. According to [17] proposed the neutrosophic versions of the Uniform, Exponential, and Poisson distributions. Additionally, by [18] extended the classical Gamma distribution to the neutrosophic setting, demonstrating its applicability in analyzing complex data. According to [19] proposed the neutrosophic Laplace distribution and its application in modelling of NIFTY50 from Indian Stock Market. According to [20] introduced a generalization of log-logistic distribution which is particularly useful for handling indeterminate data and modeling survival and reliability in the presence of uncertainty.

According to [21] presented the neutrosophic form of the normal distribution. Further applications of neutrosophic statistics can be found in [22-24]. After going through the literature, it has been observed that no such work related to the properties of Generalized Gamma Distribution under indeterminacy and uncertainty has been carried out. To address this gap, we extended the classical GGD to develop the NNGD, which was designed to model data involving uncertainty and indeterminacies. This novel distribution effectively addresses imprecise and uncertain data. We explore the statistical properties of the NNGD, including the neutrosophic mean, variance, moments, quantiles, hazard function, entropy, and reliability function. Neutrosophic parameters are estimated using the MLE method, and a simulation study is conducted to evaluate the performance of these estimates. We apply the NNGD to estimate the reliability and lifespan of gearbox components in wind turbines within the Renewable Energy sector. Additionally, we examine special cases of the NNGD, which reveal reductions to the Neutrosophic Weibull, Gamma, Exponential and Rayleigh distributions under specific parameter conditions.

I.1. RESEARCH OBJECTIVES

The primary objective of this study is to develop and apply a Neutrosophic Generalized Gamma Distribution (NNGD) for modeling uncertain and imprecise hormone secretion data. Specifically, the study aims to:

1. Extend the Classical Generalized Gamma Distribution to a neutrosophic framework capable of handling ambiguity in biological observations.
2. Analyze the secretion patterns of prolactin (PRL), luteinizing hormone (LH), and testosterone in male Beagle dogs using the proposed NNGD model.
3. Compare the hormonal profiles between control and drug-treated groups to assess the model’s effectiveness in detecting behavioral differences in hormone release.
4. Demonstrate the superiority of the NNGD approach over traditional probabilistic modeling when dealing with uncertain physiological data.
5. Explore potential future applications of the NNGD in medical diagnostics, risk assessment, and reliability-related fields.

I.2 SIGNIFICANCE AND NOVELTY

This research introduces a Neutrosophic Generalized Gamma Distribution (NNGD) as an innovative statistical framework for modeling hormone secretion data characterized by uncertainty and imprecision. Traditional probabilistic models often assume complete data accuracy, which is rarely achievable in biological systems due to measurement variability, environmental influences, and inherent physiological fluctuations. By incorporating neutrosophic principles, the NNGD accommodates these uncertainties more effectively, leading to improved data interpretation and enhanced decision-making. The novelty of this work lies in extending the widely applied Classical Generalized Gamma Distribution into a neutrosophic domain and implementing it for the first time to evaluate the secretion dynamics of prolactin (PRL), luteinizing hormone (LH), and testosterone in Beagle dogs. Unlike conventional methods that may overlook subtle secretion differences, the NNGD enables a more accurate comparison between control and drug-treated groups and provides deeper insights consistent with clinical observations. Overall, the proposed modeling approach not only advances the statistical analysis of endocrine processes but also has the potential for broad applicability in medical diagnostics, veterinary science, risk assessment, and reliability engineering, where uncertainty is a critical factor.

II. GENERALIZED NEUTROSOPHIC GAMMA DISTRIBUTION:

A Neutrosophic continuous random variable $x_m = x_p + x_v j_m$ is said to have NNGD if it has the following probability density function

$$f(x_m; \omega_m, \sigma_m, \tau_m) = \frac{\tau_m \omega_m}{\sigma_m \Gamma_{\tau_m}(\frac{\omega_m}{\tau_m})} (\frac{x_m}{\tau_m})^{\omega_m - 1} e^{-(\frac{x_m}{\tau_m})^{\tau_m}} \quad x_m > 0 \tag{2.1}$$

Here

- $\omega_m = \omega_p + \omega_v j_m > 0$ is the neutrosophic shape parameter,
- $\sigma_m = \sigma_p + \sigma_v j_m > 0$ is the neutrosophic scale parameter,
- $\tau_m = \tau_p + \tau_v j_m > 0$ is the additional neutrosophic shape parameter,

Γ is the gamma function.
 $j_m \in (0, 1)$ is the indeterminacy in parameters and variables
 The cumulative distribution function (CDF) of NNGD is given by

$$F[x_m] = \int_0^{x_m} f(x_m; \omega_m, \sigma_m, \tau_m) dx_m$$

$$\int_0^{x_m} \frac{\tau_m}{\sigma_m \Gamma \frac{\omega_m}{\tau_m}} \left(\frac{x_m}{\tau_m}\right)^{\omega_m-1} e^{-\left(\frac{x_m}{\tau_m}\right)^{\tau_m}} dx_m$$

$$F[x_m] = \frac{\gamma\left(\frac{\omega_m}{\tau_m}\right) \left(\frac{x_m}{\tau_m}\right)^{\tau_m}}{\Gamma \frac{\omega_m}{\tau_m}}$$

Where $\Gamma \frac{\omega_m}{\tau_m}$ = Complete Gamma functions.

III. MATERIALS AND METHODS

Prolactin (PRL) is a single-chain peptide hormone consisting of approximately 200 amino acids, produced by lactotropic cells in the anterior pituitary [25]. Its primary role is to promote mammary gland development and initiate lactation. In addition, prolactin is involved in around 300 biological, physiological, and homeostatic functions in the body [26], [27]. Beyond its role in regulating glucose, insulin, and lipid metabolism, prolactin also contributes to maintaining a positive energy balance [28]. Its secretion occurs in a pulsatile manner, likely influenced by multiple hypothalamic factors. Although dopamine typically inhibits prolactin release, the text states that it stimulates it, which may be a mistake [29], [30]. Luteinizing hormone (LH), also produced by the anterior pituitary through gonadotropic cells, is regulated by gonadotropin-releasing hormone (GnRH) [31]. In females, a surge in LH triggers ovulation. In males, LH stimulates Leydig cells to produce testosterone, which is crucial for sperm production [32], [33]. Elevated LH levels in males can indicate testicular dysfunction [34]. Several studies have shown that both LH and testosterone levels may decrease due to negative feedback mechanisms. Prolactin, LH, and testosterone levels were monitored over a six-hour period in two dogs-D (2 years old) and G (6 years old)—both with and without medication.

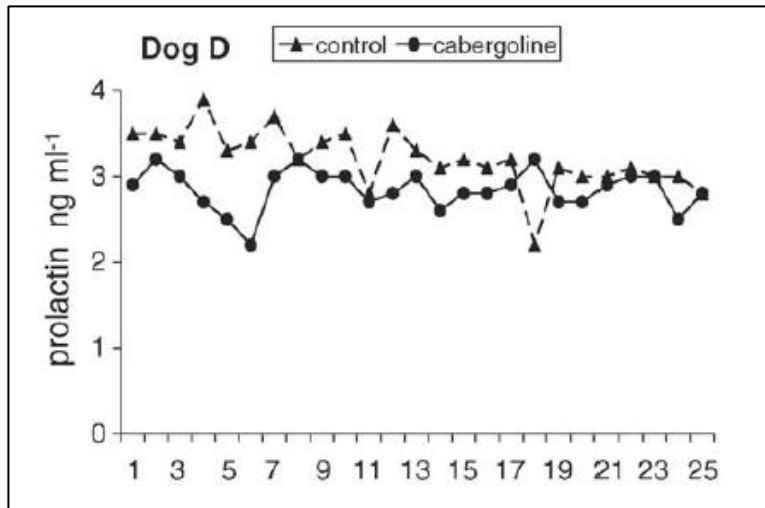


Figure 1(A): depicts the PRL level of an individual Dog D in a blood sample collected at 15-minute intervals with and without treatment.

Source: Authors, (2026).

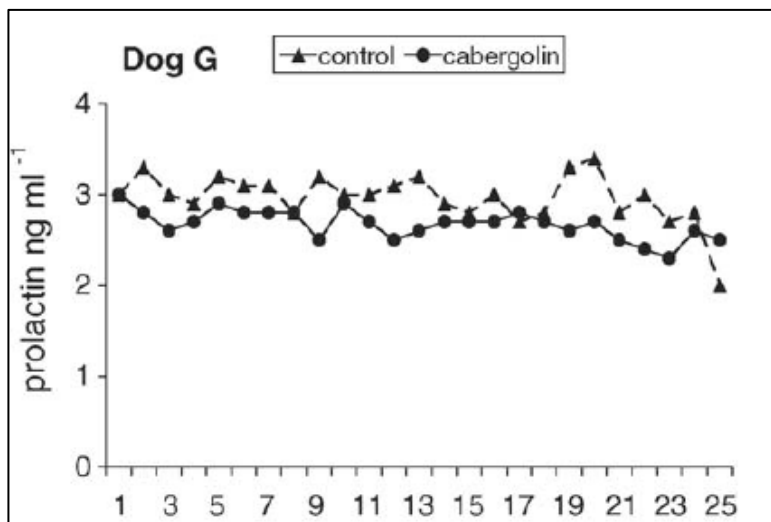


Figure 1(B): depicts the PRL level of an individual Dog G blood sample collected at 15-minute intervals with and without treatment.

Source: Authors, (2026).

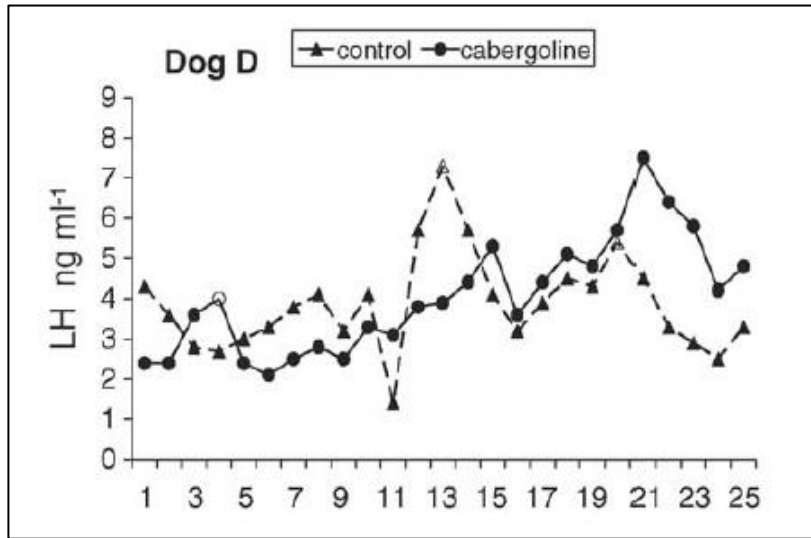


Figure 2(A): illustrates the LH level of an individual Dog D blood sample collected at 15-minute intervals under control group and cabergoline administration.
Source: Authors, (2026).

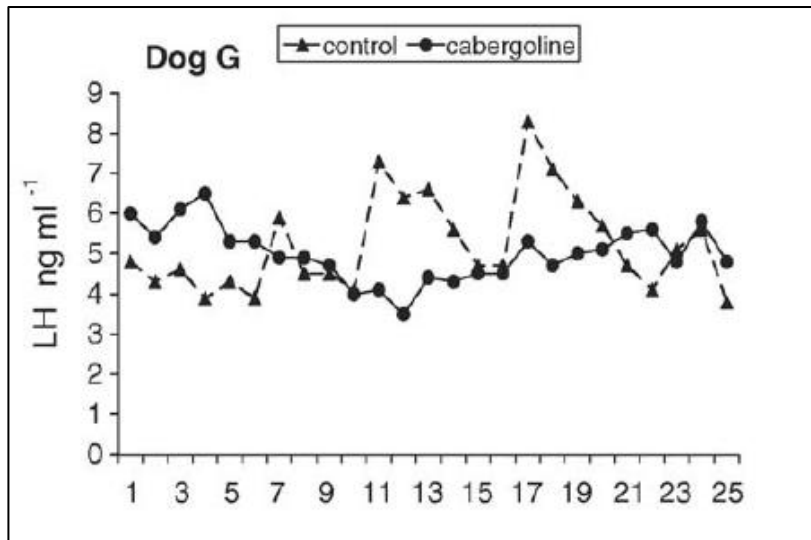


Figure 2(B): illustrates the LH level of an individual Dog G blood sample collected at 15-minute intervals under control group and cabergoline administration.
Source: Authors, (2026).

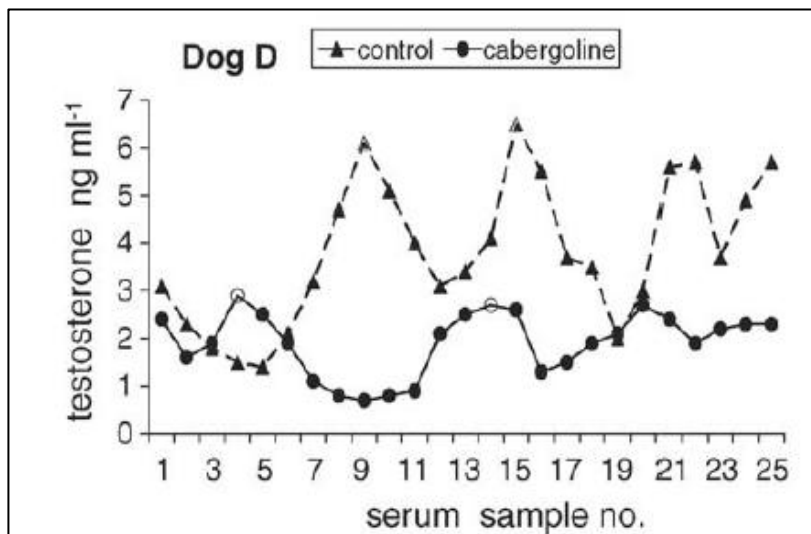


Figure 3(A): demonstrate the testosterone level of an individual Dog D blood sample collected at 15-minute intervals under control group and cabergoline administration.
Source: Authors, (2026).

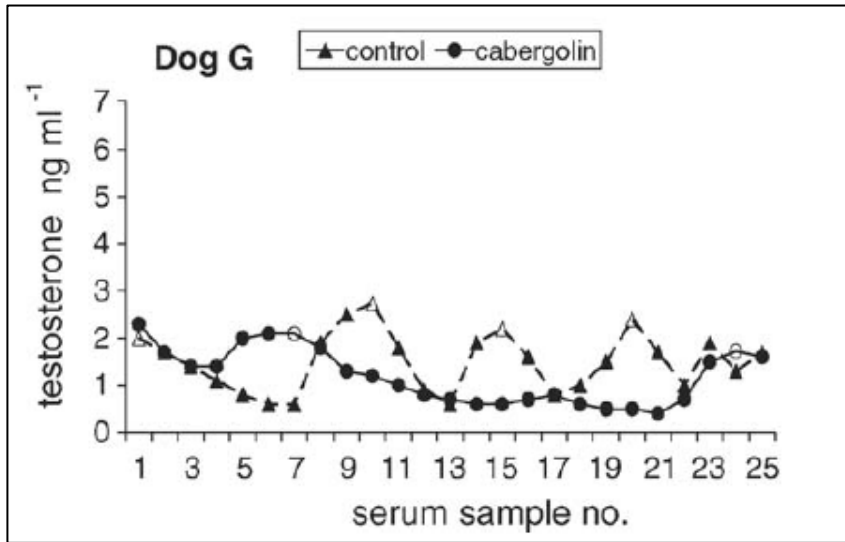


Figure 3(B): demonstrate the testosterone level of an individual Dog G blood sample collected at 15-minute intervals under control group and cabergoline administration.

Source: Authors, (2026).

The medical graph showed that serum prolactin levels in dogs D and G were likely influenced by cabergoline treatment. LH and testosterone levels remained similar before and after medication in both dogs.

IV. MATHEMATICAL RESULTS AND DISCUSSION

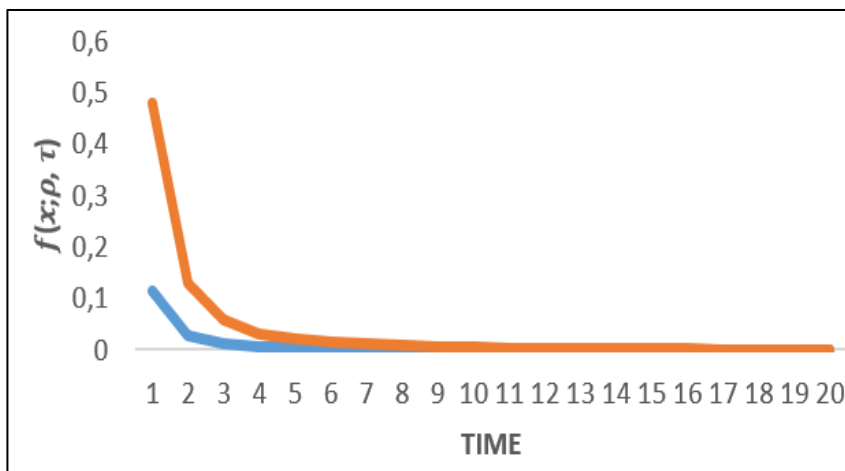


Figure 4(A): PRL Secretion Link Between Control and Medication in Dog D.

Source: Authors, (2026).

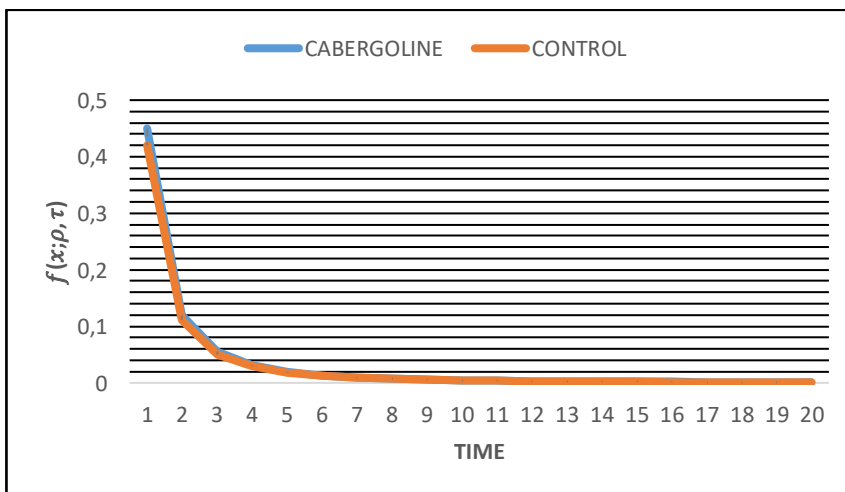


Figure 4(B): of mathematical model shows that the control and medication group are likely related to each other in secretion of PRL in the sample of dog G.

Source: Authors, (2026).

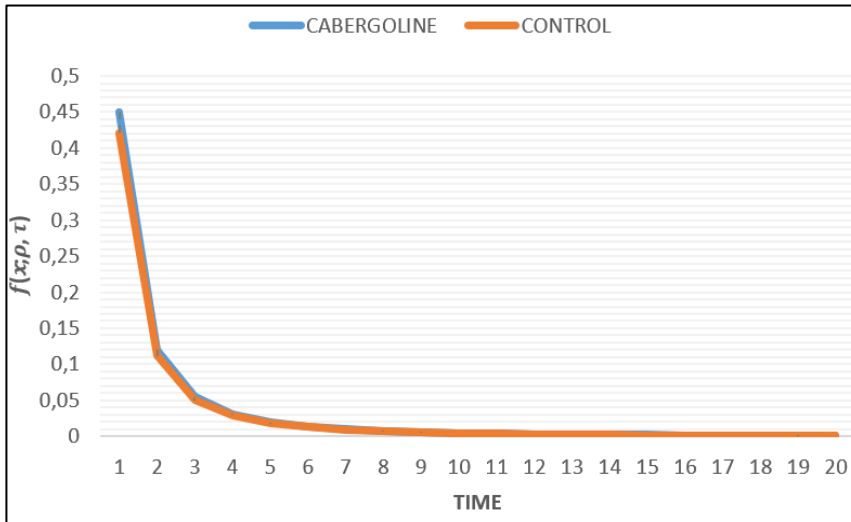


Figure 5(A): of mathematical model shows that the both group namely control and medication group are related to each other in secretion of LH in the serum sample of dog D.
Source: Authors, (2026).

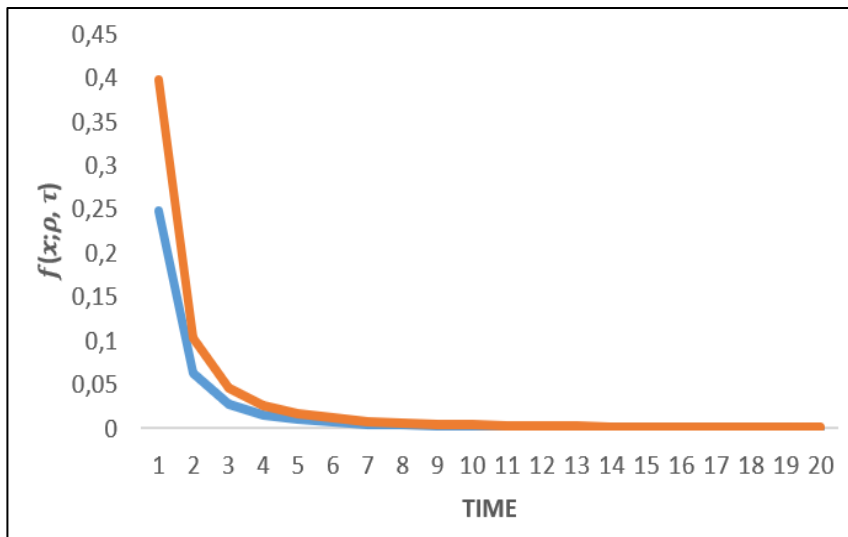


Figure 5(B): of mathematical model shows that the both group namely control and medication group are related to each other in secretion of LH in the serum sample of dog G.
Source: Authors, (2026).

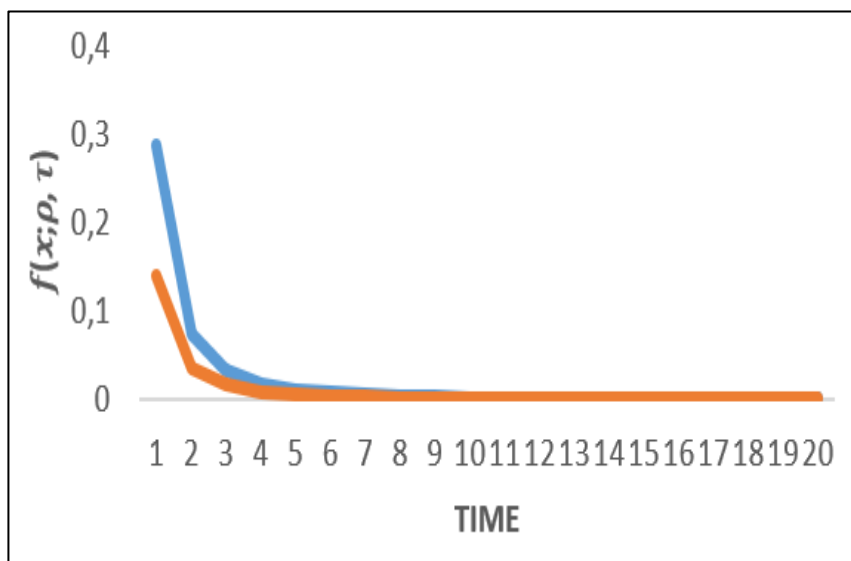


Figure 6(A): of mathematical model shows that the testosterone level in the serum sample of Dog D is significant before and after medication.
Source: Authors, (2026).

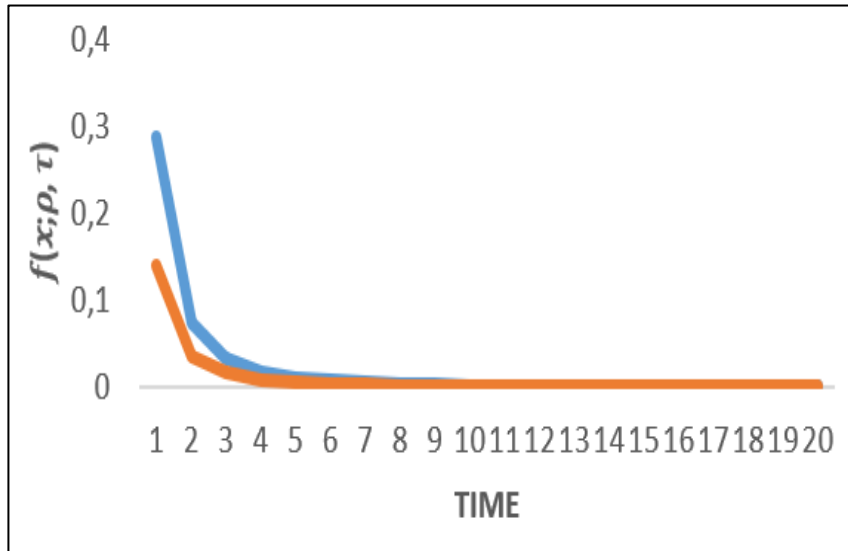


Figure 6(B): of mathematical model shows that the testosterone level in the serum sample of Dog G is significant before and after medication.

Source: Authors, (2026).

The mathematical model graph indicates a significant relationship in PRL, LH, and testosterone levels before and after medication in both dog's D and G.

V. CONCLUSION

The Neutrosophic Generalized Gamma Distribution (NGGD) effectively extends the traditional GGD by incorporating neutrosophic elements to handle uncertainty and indeterminacy, making it well-suited for complex, real-world scenarios. Simulation results show that NGGD offers improved parameter estimation as sample sizes grow, and its flexibility is highlighted by its special cases, such as the Neutrosophic Weibull, Gamma, Exponential, and Rayleigh distributions. This makes NGGD particularly useful for predictive maintenance, failure analysis, and other applications where uncertainty plays a critical role, such as engineering, manufacturing, and urban planning. In connection with the medical study, the NGGD was applied to model the secretion patterns of prolactin (PRL), luteinizing hormone (LH), and testosterone in male dogs. PRL secretion in dogs appeared to follow a non-pulsatile, constitutive pattern, differing from the combined pulsatile and non-pulsatile release seen in humans. The probability density functions generated for PRL, LH, and testosterone using the NGGD allowed for a more accurate comparison between the control and drug-treated groups. These mathematical findings align with medical observations, enhancing the understanding of hormone behavior under treatment and offering a robust framework for analyzing biological variability under uncertainty. Future work could expand this approach for applications in risk assessment, quality control, and supply chain management.

VI. AUTHOR'S CONTRIBUTION

Conceptualization: Bevara Kondala Rao, Biplab Kumar Rath and A. Manickam

Methodology: Bevara Kondala Rao

Investigation: Biplab Kumar Rath

Discussion of results: All authors.

Writing – Original Draft: All authors.

Writing – Review and Editing: All authors.

Resources: A. Manickam, Biplab Kumar Rath

Supervision: A. Manickam, Biplab Kumar Rath

Approval of the final text: All authors.

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