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SCALE-UP BY MATHEMATICAL MODELING OF ANAEROBIC REACTORS FOR YEAST WASTEWATER TREATMENT

Gabriel Alejandro Iglesias Barreto*[1] and Iván Leandro Rodríguez Rico[2]

[1] LABIOFAM VC Bioproducts Complex. Santa Clara, Cuba.
[2] Central University "Marta Abreu" of Las Villas. Santa Clara, Cuba.

[1] http://orcid.org/0000-0003-0086-2236, [2] http://orcid.org/0000-0003-1295-5368

Email: *gibarreto00@gmail.com, ivanrrico@gmail.com

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ABSTRACT

Wastewater treatment by anaerobic digestion is a viable solution for the discharge of industrial liquid wastes with different levels of contamination. The kinetics of the anaerobic digestion reaction, in laboratory tubular reactors, by a consortium of microorganisms to treat liquid waste from a feed yeast industry does not adjust to a Monod equation, but to a numerical adjustment carried out with the Table Curve 2D program. Where 0.65 d^{-1} was determined as the maximum cell growth rate and other parameters of the microbial kinetics. The error with which the selected model describes the experimental results is ± 24 mg/L. The mathematical analysis of the models used was carried out in MATLAB® software, which allowed the determination of a novel dimensionless number, named Gai, which facilitated the analysis, scaling by mathematical modeling and dimensioning of the anaerobic reactors at industrial scale for the treatment of 430 m^3/d of waste. Tubular bioreactors with a total height of 18,93 m and 2,4 m in diameter were dimensioned. The required flow rate of 2,91 m/h was also determined. The Gai number is a relationship between the governing phenomena of the studied process, the conversion and residence time constants, with two main mechanisms involved in the process: convective flow and conversion. Once it has been demonstrated that there are no diffusional restrictions in the anaerobic leaf.

I. INTRODUCTION

The context of research and technological innovations for process industry does not escape from problems involved in converting the results achieved in the laboratory into a safe and economically feasible structure for industrial scale-up. The scale-up of chemical operations represents significant challenges in terms of the economic management of the inputs involved and the acquisition of the necessary equipment, which are very different from the resources and techniques used in the laboratory for the same purpose [1] and [2]. Scaling becomes an important factor for the insertion of new production technologies originating from laboratory-scale research, which, in order to find an advantageous position in the industry, must overcome the uncertainties that arise with the change of production scale and, even if these obstacles are overcome, they do not ensure the success of an industrial plant without first developing the studies corresponding to the economic and commercial profile, the environmental impact of the proposed installation and the real availability of the materials necessary to carry out and sustain the manufacturing activity over time.

Mathematical simulation of processes is a highly effective tool for the analysis of real systems described by mathematical models obtained and validated for the description of the phenomena studied. This tool makes it possible to evaluate the behavior of the system under study. The degree of specificity and robustness of the response offered by the mathematical model depends on the rigor with which the researcher has developed it [1-3]. A great advantage of mathematical models is that it's are a very valuable tool when you want to observe the behavior of a system, but are unwilling or unable to experiment with scaling equipment,

particularly with reactors, because of the implications for process safety and feasibility [2] and [3]. Mathematical modeling is a powerful tool that can be used to understand and predict the behavior of biological systems. In the case of biological reactions, mathematical modeling can be used to describe cell growth, chemical production, and substrate degradation [4]. Mathematical models of biological reactions are usually systems of differential equations that describe the evolution of reactant and product concentrations over time. These models can be used to study the effect of factors such as temperature, nutrient concentration and the presence of inhibitors on reaction behavior. Mathematical modeling has a number of proven advantages over traditional experimental methods for studying biological reactions [5-7]. The development of mathematical models for industrial microbiological processes is an active field of research. The development of useful mathematical models must take into account factors specific to biological reactions involving microbial growth and the dependence of this multiplication on pH, temperature and nutrient availability. [8] Industrial microbiological processes usually take place in heterogeneous systems, where physical and chemical conditions may vary in space and time. The mathematical model must take this heterogeneity into account in order to faithfully predict the behavior of the process at a change of scale.

Bioreactor scale-up is a complex process that requires the consideration of a number of criteria and techniques to ensure the reproduction of microbiological reactions in industry or pilot plant scale equipment, performed and stabilized in the laboratory environment. Kossen [9] reports the fundamental methods for bioreactor scale-up with current application [10] and [11]: the fundamental method (solution of microbalances for mass, heat and momentum transfer), the semi-fundamental method (solution of simplified balances), the approximate calculation (known as rule of thumb), the trial and error with iterations and the scale-up by dimensional analysis. Dimensional analysis is a technique that performs the mathematical adjustments to vary the operating parameters at different scales in such a way that the most significant dimensionless groups or models remain unchanged when passing from one stage to another of the scaling. The physical significance of these groups is a ratio of time constants for the different mechanisms involved ensuring that the relative significance of the mechanisms does not change with increasing equipment size. This method is very efficient, but has limitations such as the impossibility of maintaining constant all the dimensionless groups, therefore, the most transcendental ones must be determined and the rest must be ignored as a result of the regime analysis. The criteria for scaling biological reactors can be divided into two main categories: physical criteria referring to the physical parameters linked to the reactor properties and operating variables, and biological criteria associated with the properties of the microbial culture used, its stability and reaction kinetics [12] and [13]. For bioreactors scale-up the most commonly used criteria are: reactor geometry, oxygen transfer coefficient if the reaction is aerobic (kLa), maximum exertion, power per unit volume (P/V), gas volumetric flow per unit volume of liquid (vvm) and gas surface velocity [14]. Current trends for the scale-up of tubular reactors for anaerobic digestion and biogas production focus on the development of larger and more efficient reactors [15]. In terms of size, larger capacity tubular reactors allow increasing biogas production and reducing investment costs. On the other hand, raising the efficiency of anaerobic digestion operation of liquid waste is focused on research and continuous improvement of mass and heat transfer mechanisms to increase biogas productivity and purification. Successful scale-up of biological reactors is essential for the development of the modern transformative process industry, committed to responsible environmental management and economically feasible [16].

The treatment of wastewater with biological processes, such as anaerobic digestion, is a viable solution to the discharge of liquid waste with different levels of chemical and biological contamination, while generating a highly valuable product from the energy point of view, biogas. The present research deal with the wastewater treatment of a feed yeast factory for animal feed, adjusting the strategy outlined for the management of liquid waste with the requirements for discharge into the environment stipulated by NC 27:2012 [17]. Liquid effluents from similar industries include the cleaning water from the installed equipment and the residue from the fermentation process. These residuals as described by Figueroa [18] present high concentrations of dissolved organic matter, nitrogen and phosphorus and a chemical oxygen demand (COD) between 50 and 20 000 mg/L. It is extremely important that these wastewaters are adequately treated before discharge into the environment to minimize their ecological impact. In order to design a system for wastewater management, process simulation tools are used; starting from laboratory scale experiences emulating the removal of COD from liquid waste by anaerobic digestion (producing biogas as a highly valuable by-product due to its energetic power) of settled sludge in treatment plants of similar industries. Biogas generation brings economic attractiveness to industrial waste management, being possible its capture, treatment and distribution for domestic or industrial use as a substitute for liquefied petroleum gas according to technologies proposed by [19] and [20]. The mathematical models obtained and validated for the anaerobic digestion carried out in the laboratory bioreactors allow evaluating the possibilities of scaling up the process to pilot and industrial plant scale dimensions. The mathematical simulation incorporates solid criteria for the design of the proposed technology, allowing to evaluate the behavior of the system under variation of operational and constructive parameters that provide valuable data for the management of the project from its conceptualization.

Obtaining the mathematical model and parameters for the simulation of the yeast plant wastewater treatment process aims to establish a methodology for scaling this operation safely and accurately with the use of computational utilities; expanding the opportunities for the development of similar technologies with the combination of methods for reactor scaling.

The main objective of this research is the study at laboratory scale of the removal of contaminants in wastewater from a feed yeast industry by anaerobic digestion in tubular bioreactors and the scaling of the same maintaining geometric similarity and using mathematical modeling, for the dimensioning of the reactors at industrial scale.

As an important result of the geometric similarity scaling and mathematical modeling of anaerobic bioreactors for the treatment of wastewater from the production of feed yeast, a dimensionless number is deduced for the system studied. This dimensionless group facilitates the analysis, scaling and dimensioning of anaerobic reactors on an industrial scale.

II. MATERIALS AND METHODS

II.1 LABORATORY STUDIES

The data necessary for the scaling of the biological treatment studied are based on a laboratory-scale study of COD removal. The treatment is carried out by a microbial consortium of sludge extracted from a wastewater treatment plant. The studies

were carried out in reactors that can be considered hybrids since they combine characteristics of traditional UASB reactors and tubular reactors [20] and [21]. For the study of the anaerobic digestion of the yeast production plant waste, a system of two tubular reactors connected in series with possibilities for axial sampling and between both reactors is used.

The treated fluid is pumped by a Masterflex L/S 7535-04 multichannel peristaltic pump with low pressure and constant flow rate of 595 ml/min based on preliminary studies by [22]. Figure 1 graphically depicts the components of the study system. The selected tubular reactors consist of two high-density polyethylene (HDPE) plastic pipes with 1,50 m length each for a total height of 3,00 m and inner diameter equal to 0,38 m; the thickness of the pipes is 2,0 mm. The selection of these materials is made according to the fluid and flow to be treated, which has a viscosity close to water's at normal temperature and pressure. To support the microbial population used for the anaerobic digestion of the waste, non-degradable commercial plastic rope, sterilized by boiling in water, was used. The filler was arranged longitudinally, occupying the inner volume of the reactor body, offering support to the microorganisms and hydraulic resistance to the flow.

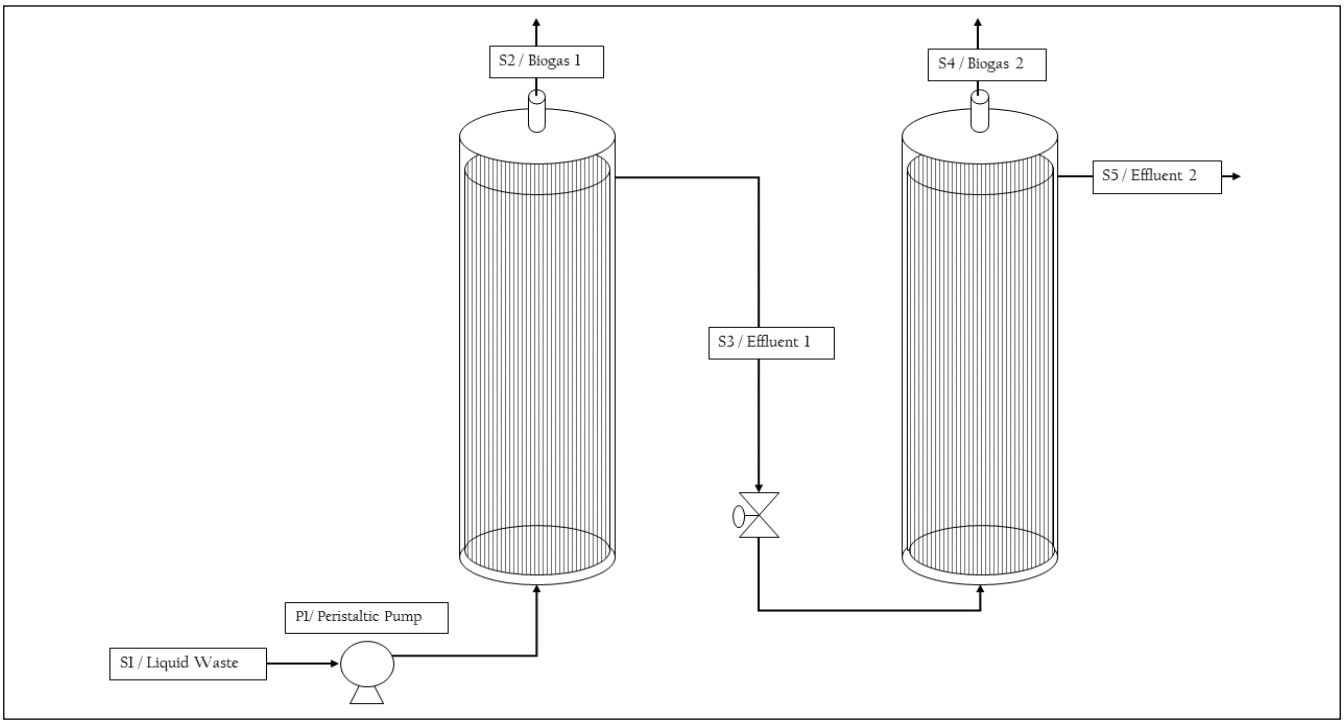

Figure 1: Schematic bioreactors used at the laboratory scale.
Source: Authors, (2023).

For the kinetic study of the microbiological reaction that develops, the measurement of COD depletion and the increase in the presence of volatile suspended solids (VSS) is proposed according to the experimental methodology described in [23]. These parameters are associated with the progress of wastewater remediation and cell biomass growth in the treatment medium respectively. On the other hand, the increase in the concentration of biogas generated, according to the technique referred to in Fernandez [24], indicates the production of this compound and allows estimating the efficiency of this treatment for its subsequent scaling up. For the measurements, multiple samples were taken along the reactors and at the exit of the first one with periods of three days of reaction until completing 45 days. The experimentally obtained data were mathematically adjusted with Table Curve 2D software [25].

II.2 MATHEMATICAL MODELLING AND SIMULATION OF THE ANAEROBIC PROCESS

The reaction yield parameter is studied as a function of the ratio biomass(X)/substrate(S) according to equation (1).

$$Y_{X/S} = -\frac{\Delta X}{\Delta S} = \frac{X_1 - X_0}{S_0 - S_1} \quad (1)$$

Instead, the cell growth rate (μ) is investigated following the relationship X=f(t) reported by the graph plotted in Table Curve 2D. The derivative of the expression fitted and evaluated for the different experimental time intervals responds then to the μ of the studied system according to equation (2).

$$\mu = \frac{1}{X}\frac{dX}{dt} \quad (2)$$

We then proceed according to the Lineweaver-Burk method applied by [26] for the linearization of the cell growth data with the Monod equation. Another usable alternative is the numerical adjustment of the data obtained from μ versus S. For this purpose, the Table Curve 2D modeling tool is used, which presents the adjustment of the experimental data with a bank of mathematical expressions in pursuit of the best possible fit to the behavior of the μ variable.

In order to choose among the models for the selection of the most adequate one as a response to the microbial kinetics of the reaction, the criterion proposed by [14] and [27] is followed. The equation that best reflects the process is the one that, when used in the simulation of the process, according to the mathematical model of a tubular reactor, produces the smallest deviations with respect to the experimental values. The deviations are estimated as the

quotient between the square of the difference between the experimental value and the magnitude calculated by the model and the square of the experimental value for each variable as expressed in equation (3). Although this deviation is determined for the variables substrate concentration, biomass and an overall deviation, the behavior of the substrate concentration variable is of special attention, since the objective of the process is the decontamination of the residual.

$$\varepsilon = \frac{\left(X_{exp} - X_{cal}\right)^2}{X_{exp}{}^2} \quad (3)$$

To validate the selected mathematical model corresponding to the cellular kinetics of the process, the function obtained is subjected to mathematical analysis criteria. Starting with the domain analysis and physical restrictions, sign analysis, monotony and concavity of the same starting from the sign analysis of the first and second derivatives. The maximum velocity value is an important parameter in the study of kinetics, in addition to the fact that it can be contrasted with the data documented in Dunn [28] to classify the biological composition of the microbial population used in the experiment.

The substrate removal at laboratory scale responds to equation (4) studied by [29] and [30], where the parameter μ is replaced by the mathematical model obtained according to the methodology of section II.2. Where the term γ includes properties of the bioleaf and microbial growth, remaining approximately constant at 10 409 mg/L, with variations in the flow rate to be treated [14]. This value of γ is considered small, since much higher biomass concentrations in bioleafs are reported up to 20 000 mg/L as documented in [28]. The variable v responds to the flow rate, μ the selected expression of cell growth rate and the calculated Yx/s yield.

$$\frac{dS}{dZ} = -\frac{1}{v} \cdot \frac{\mu}{Y_{X/S}} \cdot \Upsilon \quad (4)$$

The mathematical development of this expression as a function of the growth rate equation for substrate variations can be simulated in the MATLAB® software suite [31] to inspect the progress of the reaction with respect to the length of the reactors for different diameters and thus multiple length/diameter ratios of the reactors. At larger than laboratory scale in either pilot or industrial plants it is necessary to include the effectiveness factor (equation 5) in the expression (4) for substrate removal.

$$\eta = \frac{r_{S\,cal}}{r_s} \quad (5)$$

Based on these considerations, equation (6) can be used as a mathematical model for substrate removal, taking z as the useful height of the reactor, the expression of the effectiveness can be evaluated for flow rate values that cause a decrease in DOC from a value S_0 to a value S measured experimentally.

$$\eta = \frac{\int_{S_0}^{S} \frac{dS}{\mu}}{\Upsilon \cdot z} \cdot v \cdot Y_{\frac{X}{S}} \quad (6)$$

II.3 PROCESS SCALING

With the interest of scaling up the process to industrial plant dimensions, a methodology is proposed to design biological reactors for the anaerobic digestion of 430 m^3/d of the wastewater from the feed yeast industry, where the levels of COD removal and biogas production obtained at laboratory scale can be reached.

For the scale-up from the laboratory to an industrial scale, the geometric similarity between both scales and mathematical modeling are used, based on a dimensionless number developed during the research. Taking into account the geometrical similarity of the reactors, based on the ratio length/diameter (L/D) of the laboratory reactors equal to 7,89, it is possible to estimate the dimensions of a reactor for the treatment of this type of waste with a waste volume 490 times higher than the one installed on a small scale according to equation (7).

$$V = \frac{\Pi \cdot D^2}{4} \cdot L \quad (7)$$

The mathematical model developed relates the dimensionless number and the removal of S, a fundamental aspect for sizing the industrial equipment for wastewater remediation.

The simulation with the mathematical models and the estimation of the error, facilitate the validation of the results to be obtained, offering a methodology for the calculation of the design dimensions of the anaerobic digestion reactors.

III. RESULTS AND DISCUSSIONS

III.1 MATHEMATICAL MODELATION AND SIMULATION

The experimental data were plotted using Table Curve 2D software, the graphs obtained are shown in the following Figures where the behavior of the parameters COD, VSS and biogas production with time progress is verified. Figure 2 describes the drop of COD with the progress of the anaerobic digestion in the laboratory where it can be seen that after 25 days of reaction, the demand remains practically unchanged asymptotically for more than 2 000 mgO_2/L of treated effluent.

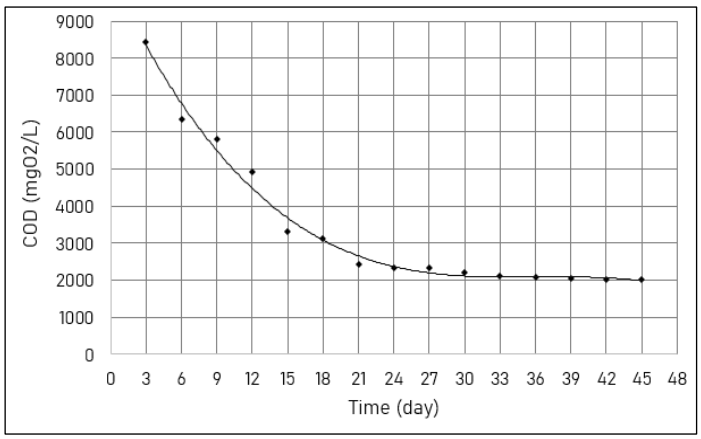

Figure 2: COD variation over time.
Source: Authors, (2023).

Figure 3 shows the appearance of volatile suspended solids with advancing reaction time associated with cell multiplication which is interpreted as exponentially increasing over the entire domain of the experiments performed for a practically constant Yx/s yield of 0,18. These data are in agreement with previous studies performed by Simeonov [32].

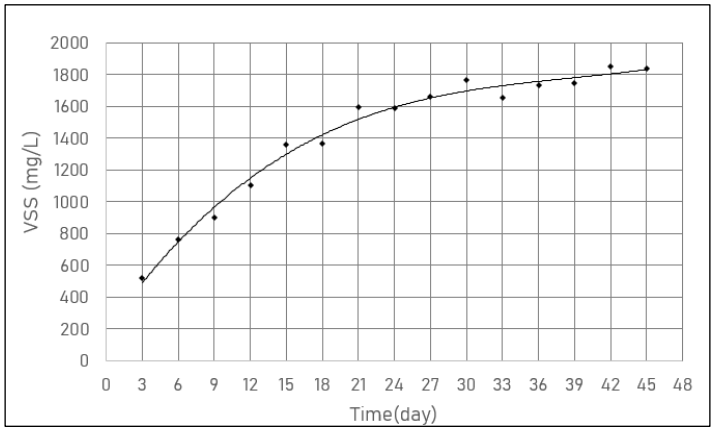

Figure 3: VSS variation over time.
Source: Authors, (2023).

The production of biogas as a high value-added co-product of anaerobic digestion of liquid waste is shown in Figure 4 with the increase in biogas concentration as the reaction time progresses. This behavior shows promising results in terms of the selected technological alternative generating 3 600 mg of biogas per liter of treated liquid for a reaction time of 21 days, obtaining a maximum production in the order of 5 400 mg/L for a final time of 45 days of digestion. The biomass production yield calculated according to equation 1 resulted in an almost constant value of 0,18, consistent with the specialized literature published in [33] for similar studies.

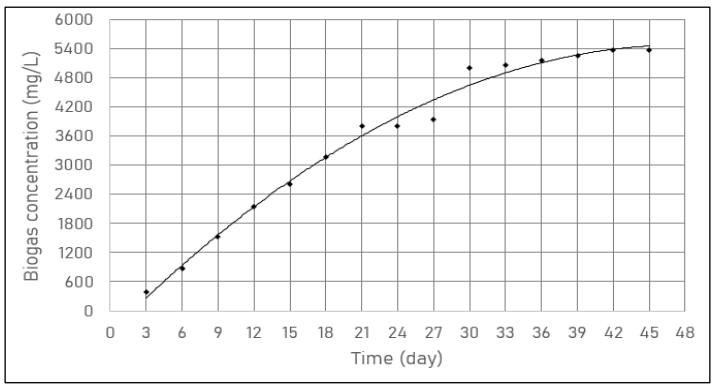

Figure 4: Biogas concentration over time.
Source: Authors, (2023).

The adjustment of the experimental data for cell growth according to the Monod equation is shown in Figure 5. It is observed that the microbial development of the studied population does not fit the Lineweaver-Burk linearization technique.

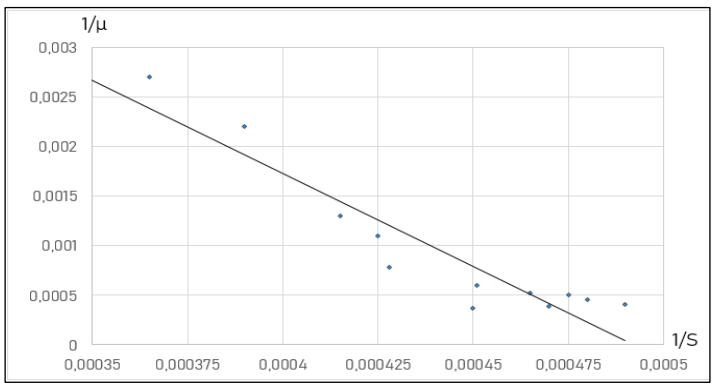

Figure 5: Lineweaver-Burk linearization.
Source: Authors, (2023).

Therefore, for the modeling of this process a numerical adjustment of the variable μ and the depletion of S is explored with the Table Curve 2D tabulator tool, obtaining among others the expressions related in Table 1.

Table 1: Numerical adjustments of μ=f(S).

Table Curve 2D equation	Equation	Parameters	R^2
80	$\mu = \left(a + \frac{b}{S}\right)^2$	b = -1158,82 a = 0,529	0,97
78	$\mu = \left(a + \frac{b}{\sqrt{S}}\right)^2$	b = -36,37 a = 0,798	0,954
38	$\mu = \exp\left(a + \frac{b}{S}\right)$	b = -11118,68 a = -0,429	0,98
39	$\mu = \exp\left(a + \frac{b}{S^{1,5}}\right)$	b = -497660,34 a = -1,139	0,96

Source: Authors, (2023).

Following the criteria for the selection of the kinetic model exposed in section II.2, the expression 38 of Table 1 whose coefficient of determination adjusts for 98% being superior to the rest of the expressions. The graphical performance of this equation and its contrast with the experimental measurements is shown in Figure 6.

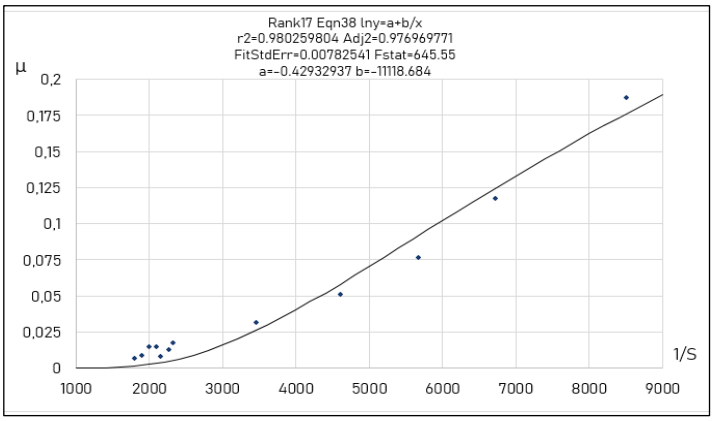

Figure 6: Cell growth rate versus substrate depletion.
Source: Authors, (2023).

The domain of expression chosen for the description of cell growth kinetics is all positive non-zero real substrate concentration S values. This mathematical restriction obeys the logic that it does not make sense to assume values of S less than or equal to zero since the amount of substrate to be administered to the biological system is not determined by negative or zero values, leaving the equation undefined for these values. On the other hand, the behavior of too small substrate concentration values closes to the proximity of zero is analyzed by applying the limit criterion for the independent variable S. In this case, given the restriction imposed S>0, only the case described in equation (8) is of interest. The result is logical and indicates that as the substrate is depleted the rate of reaction slows down until it stops.

$$\lim_{S \to 0} \exp\left(a + \frac{b}{S}\right) = 0 \tag{8}$$

On the other hand, the behavior when the substrate concentration is much higher approaching infinity indicates the existence of a possible horizontal asymptote that responds to the linear equation y=mS+b, defined in equation (9) whose intercept with the axis of the ordinates in the b parameter.

$$m = \lim_{S \to +\infty} \frac{\exp\left(a + \frac{b}{S}\right)}{S} = 0 \quad (9)$$

$$b = \lim_{S \to -\infty} \left(exp\left(a + \frac{b}{S}\right) - 0 \cdot S\right) = \exp a \quad (10)$$

These results verify that the cell growth equation presents a horizontal asymptote (equation 10) for when S is in excess, μ tends to the value of 0.65 d^{-1}, as the reaction rate will not increase after this value is considered the same as the maximum growth rate of this biological system (μ_{max}) calculated in (11).

$$y = \exp(-0{,}429) = 0{,}65 \quad (11)$$

The analysis of the sign of the function indicates that μ>0 for all values of S defined in the previous analysis. The first derivative of the function indicates that μ is increasing throughout its domain since b is always less than zero and therefore, there is no change in the sign of the first derivative around this point. The second derivative reports that at the point for S equal to half the parameter b, the function has a change of concavity from convex to concave. Indicating that the slope of the curve increases progressively and then begins to decrease corroborating the asymptotic behavior at μ_{max}. The value of b can be taken as an indicator of the affinity. For a lower modular value of b, the change in concavity would occur faster and therefore indicates that the microorganisms would consume the substrate better. Based on the above demonstrations it is convenient to express the kinetics of cell growth according to equation (12).

$$\mu = 0.65 exp\left(\frac{-11118{,}68}{S}\right) \quad (12)$$

Where a = 0,65 d^{-1} which is interpreted as the maximum value that μ can reach; b=-11118,68 mg/L; μ_{max}=0,65d^{-1}. This maximum growth rate value is between the values of μmax reported for populations of methanogenic and acidogenic microorganisms reported by [28], although it is not specifically that of either of these two groups, which is due to the fact that a mixed population of digesting microorganisms is being analyzed.

The computational tools used to explore the mathematical model obtained for the dimensioning of the bioreactors offer conclusive results for the experiences studied. It provides the calculations of diameter and length of the bioreactor vessels at laboratory and pilot plant scale, reporting 0,40 m and 1,03 m in diameter; on the other hand, the calculated heights were 2,56 m and 4,96 m for each scale, respectively. We proceeded to the simulation of this model to determine the accuracy with which it describes the process at laboratory scale, summarizing in Figure 7 the results plotted by the MATLAB® software for different flows of the wastewater to be treated, linked to possible relations of the reactor dimensions if this parameter were kept fixed or depending on the availability of the materials to be accessed in future experiments. The error with which the studied model describes the experimental results at this scale of work is ± 24 mg/L being acceptable.

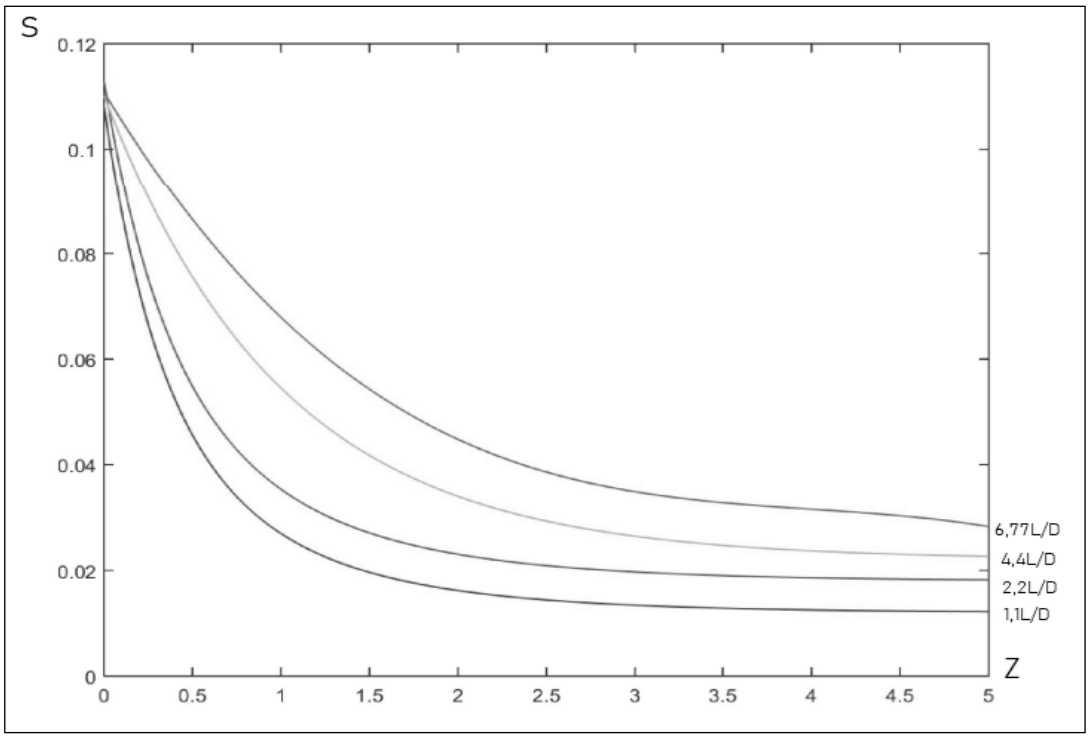

Figure 7: Mathematical simulation of substrate depletion as a function of reactor length.
Source: Authors, (2023).

The modeling at Pilot Plant scale was performed using the mathematical considerations and expressions developed in section II.2 and the results are shown in Figure 7. It shows that the removal effectiveness parameter described in equation (13) is a non-unitary variable, which indicates that the interactions (in this case linked to convective transport) between cell metabolism and physical transport within the reactor are significant. The software provided the mathematical model (equation 14) that describes the relationship of the removal effectiveness and the volumetric flow rate to be treated at the pilot scale.

$$\eta = a + b \cdot v^{0{,}15} \quad (13)$$

Where, a is a dimensionless constant equal to 0,15 and b equal to 0,359 m/volume unit.

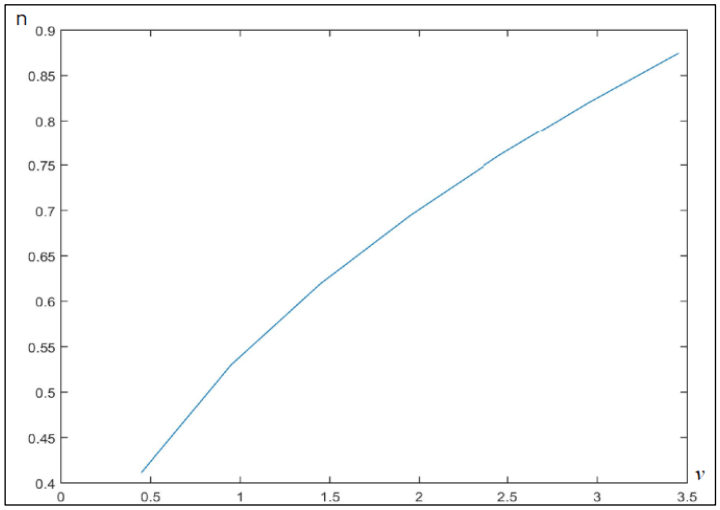

Figure 8: Removal operation effectiveness versus volumetric flow rate.
Source: Authors, (2023).

The removal of DOC in the pilot scale is proposed according to the mathematical development according to equation (14) with an estimated error of ± 71,91 mg/L, admissible according to the projected flow rates and the provisions of NC 27:2012 [4].

$$\frac{dS}{dz} = -\frac{1}{v} \cdot \frac{\mu \cdot Y}{Y_{X/S}} (a + b \cdot v^{0.5}) \quad (14)$$

III.2 PROCESS SCALE-UP

By rewriting the previous mathematical model, complying with mathematical considerations (15), (16) and (17) and integrating it to equation (14), the expression (18) is obtained, which corresponds to the dimensionless model of the process at pilot plant scale.

$$S^* = \frac{S}{S_0} \quad (15)$$

$$z^* = \frac{z}{L} \quad (16)$$

$$\varepsilon = \frac{\mu \cdot Y}{Y_{X/S}} \cdot (a + b \cdot v^{0,5}) \quad (17)$$

$$-\frac{v \cdot S_0}{L \cdot \varepsilon} \frac{dS^*}{dz^*} - \exp\left(\frac{\mu_2}{S_0 \cdot S^*}\right) = 0 \quad (18)$$

From this mathematical arrangement it is very interesting to obtain the dimensionless group (expression 19) on which the solution of the scaling problem depends and which is no more than a relation between the conversion and residence time constants. It thus reflects the two mechanisms involved in the process: convective flow and conversion, once it has been shown that there are no diffusional restrictions in the anaerobic leaf. This group is similar to the D/uL dispersion modulus, although the relationship between linear velocity (v) and maximum reactor length (L) is not directly proportional. This group presents specific parameters of the particular process under study, all of which makes it approximate a dimensionless modified Damköhler number assigned the name Gai (19). The laboratory reactor presents a Gai=0,101 which depends, as shown in (19), on the flow rate (v), the length of the reactor (L), the initial substrate concentration (S0) and the array ε enclosing the parameters of the kinetics of the cell reaction unwrapped and the removal effectiveness of the equipment.

$$Gai = \frac{v \cdot S_0}{L \cdot \varepsilon} \quad (19)$$

The development of this dimensionless number is extremely useful to accomplish the scaling of the system studied in the laboratory (subscript s) described in section II.1. The small model was used to treat 0,858 m^3/day of wastewater however, it is necessary to design a larger scale equipment (subscript L) similar to guarantee the treatment of the waste in a satisfactory operation. To assure this similarity, the length and diameter of a geometrically equal reactor (expression 20) capable of assimilating up to 430 m^3/day of the waste generated is calculated.

$$\frac{L_s}{D_s} = \frac{L_L}{D_L} = 7{,}89 \quad (20)$$

Where: s-small reactor and L- large reactor.

The solution of equation (7) considering the constraint (20) for the new volumetric flow rate leads to the dimensioning of the larger reactor with a height of 18,93 m (with an arrangement similar to the laboratory system analyzed, of two 9,46 m tubular reactors in series) and 2,4 m in diameter. Then the dimensionless group Gai must remain constant for both scales ensuring that the controlling mechanisms of the process remain the same at both scales by varying v, given the increase of the total length of the reactor and volumetric flow of influent to be received. Setting Gai=0,101 and clearing from (19) we obtain that v=2,91 m/h using the facilities of the MATLAB® mathematical package. The influence of the Gai number on substrate removal is shown in Figure 9, obtained by mathematical simulation also in the MATLAB® program.

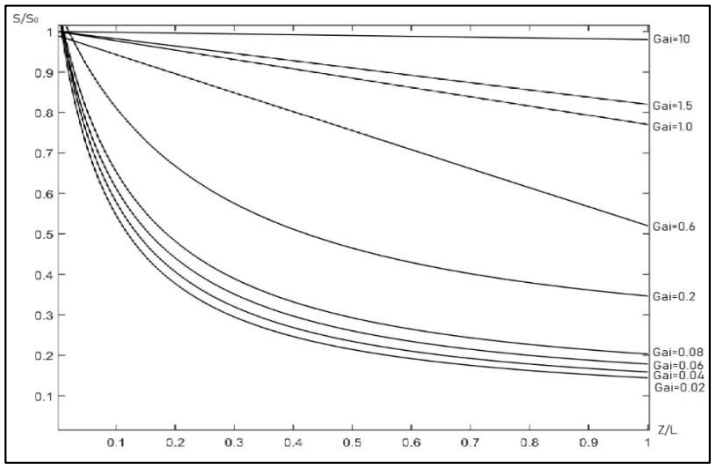

Figure 9: Influence of dimensionless number Gai on substrate removal for different lengths.
Source: Authors, (2023).

IV. CONCLUSIONS

The developed research concludes with a methodology that allows describing the anaerobic digestion of wastewater from a fodder yeast production plant. The adjustment with Table Curve 2D software and the mathematical analysis of the cell growth equation allowed the classification of the microbial population studied, characterized as a mixed culture of methanogenic and

acidogenic microorganisms. The maximum growth rate was estimated at 0,65 d^{-1} and the biomass/substrate productivity at 0,18. The resulting model has a deviation of no more than ±24 mg/L for the calculation of the cell growth rate with respect to the analytical determinations at laboratory scale. Mathematical analysis of the models with MATLAB® software led to a new dimensionless number, named Gai, ($Gai = \frac{v \cdot S_0}{L \cdot \varepsilon}$), similar to the dispersion modulus D/uL. Its development facilitated the analysis, scaling by mathematical modeling and dimensioning of industrial-scale anaerobic reactors. This group approximates a dimensionless modified Damköhler number. For the scaling, the geometric similarity (L/D) between the scales and the constancy of the dimensionless group Gai=0,101 was ensured ensuring the constancy of the reaction controlling mechanisms and the flow regime. As a result of the scaling, industrial tubular bioreactors were dimensioned in series to process a flow rate of 430 m^3/d with a total height of 18,93 m and 2,4 m diameter. The required flow speed equal to 2,91 m/h, was also determined.

V. AUTHOR'S CONTRIBUTION

Conceptualization: Iván Leandro Rodríguez Rico.
Methodology: Gabriel Alejandro Iglesias Barreto and Iván Leandro Rodríguez Rico.
Investigation: Gabriel Alejandro Iglesias Barreto and Iván Leandro Rodríguez Rico.
Discussion of results: Gabriel Alejandro Iglesias Barreto and Iván Leandro Rodríguez Rico.
Writing – Original Draft: Gabriel Alejandro Iglesias Rodríguez.
Writing – Review and Editing: Iván Leandro Rodríguez Rico.
Supervision: Iván Leandro Rodríguez Rico.
Approval of the final text: Iván Leandro Rodríguez Rico.

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VII. REFERENCES

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