



RESEARCH ARTICLE

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AN IN-DEPTH, THREE-DIMENSIONAL STRESS ANALYSIS OF THICK POROUS FUNCTIONALLY GRADED SANDWICH PLATES

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ABSTRACT

This study investigates the stress distribution in porous, functionally graded sandwich plates by combining analytical methods with three-dimensional finite element analysis. A novel higher-order displacement theory incorporating shear and normal effects is utilized. In the absence of a shear correction factor, the transverse shear stresses vanish at the upper and lower surfaces of the plate. The results highlight the influence of thickness on stress distribution and through-thickness displacement, with comparisons between models that include and exclude thickness effects. Finite element simulations are employed to verify the analytical results. The impact of key parameters, such as porosity and plate thickness, is also examined.



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I. INTRODUCTION

Functionally graded materials are materials or structural systems whose properties vary smoothly with spatial position to achieve optimal structural performance. By grading the volume fraction, morphology, and placement of the reinforcement, and by tailoring the matrix alloy (metal) and reinforcement material (ceramic), designers achieve smooth spatial changes in properties. The most basic forms of FGMs involve a continuous transition between two distinct constituents. Discrete, stepwise gradation may be used as a practical approximation, provided it is designed to emulate an underlying continuous gradient. Based on composition, FGMs are commonly categorized as metal-based or ceramic-based. Characteristics that are usually regarded as incompatible may be combined in functionally graded materials (FGMs), such as metals' hardness, strength, machinability, and bonding properties, together with heat resistance, wear resistance, and oxidation resistance, all without causing significant internal stress. The idea of improving the quality of materials by mixing components has been known since the creation of steel. In nature, bamboo exhibits a graded structure at the macroscale, increasing strength in concentrated areas. Bone, however, is generally regarded as a porous composite rather than a graded material. In the 1950s–1970s, researchers in the United States periodically investigated composite materials.

By [1] proposed mixing deposition mixtures of two phases to create man-made graded combinations of components. Later, in 1984, materials scientists in Japan formally introduced the concept of FGMs to develop heat-barrier materials. The Japanese government's Agency of Science and Technology (STA) subsequently supported initiatives to apply FGMs for improved energy conversion efficiency. By altering the material and its production method, microstructural characteristics may be varied spatially, enabling a wide range of constituent and configuration possibilities not limited to ceramic-reinforced metal matrices [2]. Sandwich structures are extensively used in engineering applications owing to their exceptional failure resistance, low density, and high bending stiffness. These structures consist of three layers: a relatively thick, flexible, and lightweight core surrounded by two thin, strong, and rigid sheets. In biomechanical, automotive, mechanical, civil, aerospace, and shipbuilding engineering, such structures provide significant benefits by [3-5].

Extensive research has been conducted on their behaviour: Ha [6] provided a finite element analysis of sandwich plates; [7] investigated local effects around inserts; [8] examined free vibration; [9] studied bending and vibration; [10] applied the generalised differential quadrature method; [11] proposed computational characterisation and experimental validation of aluminium honeycomb panels; and [12] analysed impact response. The sudden changes in stress at the edges of sandwich structures were fixed when the functionally graded material idea was introduced. This is because the characteristics of the material change all the way through its thickness. These novel materials are composed of metals mixed with ceramics or other materials. The static behavior and vibrational response of functionally graded sandwich plates have been extensively investigated. For instance, Zenkour [13] investigated the free vibration and buckling of a sandwich plate with a functionally graded surface that was simply supported. By [14] presented a novel displacement for the static response of functionally graded sandwich plates, which is grounded in high-order shear deformation theory. A novel hyperbolic shear deformation theory for the buckling and vibration of functionally graded sandwich plates was introduced by [15]. In turn [16] investigated the free vibration characteristics of sandwich plates with functionally graded layers using a four-variable refined plate theory. According to [17] investigated the free vibration of sandwich plates with functionally graded (FG) face sheets under a range of temperature conditions. The hygrothermal vibration and buckling of FGM sandwich plates sitting on Winkler-Pasternak elastic foundations were shown by [18].

Porous structures are extensively employed in aerospace, civil, and marine engineering applications. FGPMs, a type of functionally graded material, exhibit a porosity distribution that changes continuously throughout the volume. The structure of the basic material is porous and the holes are distributed in different directions. Modifications in either the size or quantity of pores can result in variations in porosity by [19]. In structural design, it is important to know how FGM constructions respond to static and dynamic loads. Several theories by [20] are offered to predict how FGM structures will respond to different workloads. Numerous researchers have studied the buckling, bending, and vibration behaviors of graded materials using both theoretical and experimental approaches. Therefore, while designing plates [21] and sandwich plates, it is crucial to consider porosity together with high-order shear-deformation theories [22]. These two aspects can be combined in modelling, but one is not derived from the other. User-defined material subroutines (UMAT) have been recently applied in ABAQUS to capture the continuous spatial variation of material characteristics in studies of the static and dynamic behavior of plates. For example, [23] applied a high-order finite element approach to analyse the dynamic behaviour of soft-core sandwich plates. By [24] studied the free vibration of annular sandwich plates on Winkler-type elastic foundations using a finite Taylor transform and a zigzag global-local plate theory. In turn [25] investigated free vibration analysis of sandwich plates with graded constituents through an improved higher-order sandwich panel theory. According to [26] examined the flexure of plates subjected to power-law regulation using ABAQUS UMAT.

By [27] analysed bending and vibration of plates with UMAT. In turn [28] performed bending analysis of thick porous sandwich plates, accounting for thickness stretching effects. According to [29] modelled static and free vibration responses of sandwich plates, while [30] extended the analysis to thermally loaded cases. By [31] used a four-parameter power law to study free vibration in ABAQUS. This study presents a novel higher-order theory that simultaneously accounts for both transverse shear and normal deformations to investigate the stress distribution in porous sandwich plates with FGPM face sheets. The distinctive feature of this theory lies in its ability to model transverse shear and normal deformations without the need for shear correction factors, while ensuring that the transverse shear stresses vanish at the plate surfaces. It incorporates a nonlinear distribution of transverse shear stresses, improving accuracy over first-order and classical shear theories. Porosity effects are incorporated independently, allowing the combined influence of gradation and porosity to be assessed more realistically. The constitutive behavior of the porous FGPM sandwich plate is modelled in ABAQUS using a FORTRAN UMAT subroutine. Analytical results are compared with numerical simulations and published experiments, validating the effectiveness and accuracy of the proposed theory.

II. THEORETICAL FORMULA

II.1 FGM SANDWICH PLATE

The square porous sigmoid FG sandwich plate is composed of three elastic layers with distinct positions through the thickness. Layer 2 is positioned between Layer 3 at the top and Layer 1 at the bottom. The layers are described within a rectangular coordinate system (x , y , z), as shown in Figure 1. For clarity, Figure 2 depicts the ratio of each layer, from the lower layer to the upper layer, is represented using a three-number combination, such as 1-0-1.

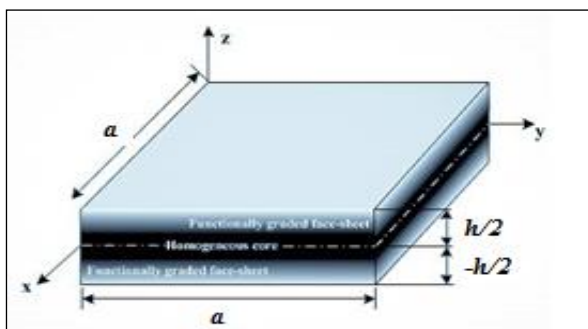


Figure 1: The FGM sandwich plate's geometry.

Source: Authors, (2026).

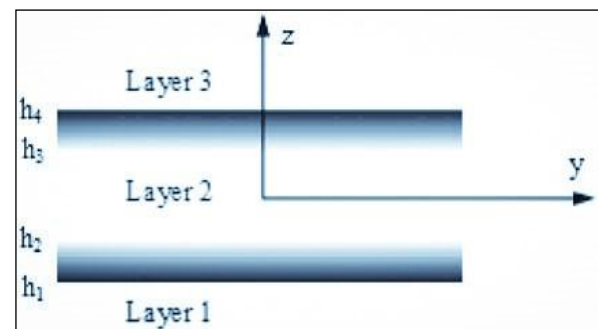


Figure 2: Layers of FGM rectangular sandwich.

Source: Authors, (2026).

A sigmoid function governs the continuous variation of the functionally graded material's volume fraction across the plate thickness.

$$V_C = \begin{cases} \left. \begin{aligned} &1/2((z-h_1)/(h_m-h_1))^k, h_1 \leq z \leq h_m \\ &1-1/2((z-h_2)/(h_m-h_2))^k, h_m \leq z \leq h_2 \end{aligned} \right\} & \text{layer 1} \\ \left. \begin{aligned} &1, h_2 \leq h \leq h_3 \end{aligned} \right\} & \text{layer 2} \\ \left. \begin{aligned} &1-1/2((z-h_3)/(h_n-h_3))^k, h_3 \leq z \leq h_n \\ &1/2((z-h_4)/(h_n-h_4))^k, h_n \leq z \leq h_4 \end{aligned} \right\} & \text{layer 3} \end{cases} \quad (1)$$

II.2 POROSITY IN FGM SANDWICH PLATES

In the SFGM sandwich plate, the non-porous core exhibits a thickness-wise variation in Young's modulus, given by the following relation:

$$\begin{aligned} E^{(1)}(z) &= (E_c - E_m) V^{(1)}(z) + E_m - \frac{\alpha}{2}(E_c + E_m) \\ E^{(2)}(z) &= (E_c - E_m) V^{(2)}(z) + E_m \\ E^{(3)}(z) &= (E_c - E_m) V^{(3)}(z) + E_m - \frac{\alpha}{2}(E_c + E_m) \end{aligned} \quad (2)$$

Where α represents the porosity coefficient ($\alpha < 1$), E_c and E_m represents Young's modulus of ceramic and metal respectively.

II.3 KINEMATICS AND CONSTITUTIVE EQUATIONS

The displacement field can be given as follows:

$$\begin{aligned} u(x, y, z) &= u_0(x, y, z) - z \frac{\partial w_b}{\partial x} + f(z) \frac{\partial w_s}{\partial x} \\ v(x, y, z) &= v_0(x, y, z) - z \frac{\partial w_b}{\partial y} + f(z) \frac{\partial w_s}{\partial y} \\ w(x, y, z) &= w_b(x, y, z) + w_s(x, y, z) + g(z) \phi(x, y, z) \end{aligned} \quad (3)$$

The transverse displacement w is composed of a displacement due to bending w_b and a displacement w_s due to shearing as well as a displacement w_{st} due to the thickness stretching effect. w_b and w_s are functions of x and y coordinates and w_{st} is a function of x , y , and z . The shape functions are in the form:

$$f(z) = z - \psi(z) \quad (4)$$

$$\psi_1(z) = zh^2 / (5h^2 + 4z^2) - 4 / 27z^3 / h^2 \quad (5)$$

$$\psi_2(z) = 5 / 4z(1 - 4 / 3z^2 / h^2) \quad (6)$$

$$\psi_3(z) = h \sin(\pi z / h) / \pi \quad (7)$$

$$g(z) = 1 - \frac{df(z)}{dz} \quad (8)$$

The strains corresponding to the displacements in Equation (3) are given by

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 + z k_x^b + f(z) k_x^s \\ \varepsilon_y^0 + z k_y^b + f(z) k_y^s \\ \frac{\partial}{\partial z} g(z) \phi(x, y) \\ \gamma_{xy}^0 + z k_{xy}^b + f(z) k_{xy}^s \\ g(z) \gamma_{yz}^s \\ g(z) \gamma_{xz}^s \end{Bmatrix} \quad (9)$$

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}, \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial^2 w_b}{\partial x^2} \\ -\frac{\partial^2 w_b}{\partial y^2} \\ -2\frac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix}, \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial^2 w_s}{\partial x^2} \\ -\frac{\partial^2 w_s}{\partial y^2} \\ -2\frac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix}, \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} = \begin{Bmatrix} \frac{\partial w_s}{\partial y} + \frac{\partial w_z}{\partial y} \\ \frac{\partial w_s}{\partial x} + \frac{\partial w_z}{\partial x} \end{Bmatrix}, \varepsilon_z = \frac{\partial}{\partial z} g(z) \varphi(x, y) \quad (10)$$

For elastic and isotropic FGMs, the constitutive relations can be written as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \quad (11)$$

with $(\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{xz}, \tau_{xy})$ and $(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{yz}, \gamma_{xz}, \gamma_{xy})$ the components of strain and stress, respectively. The three-dimensional elastic constants C_{ij} are defined as follows:

$$C_{11} = C_{22} = C_{33} = \frac{(1-\nu)}{\nu} \lambda(z), \quad C_{12} = C_{13} = C_{23} = \lambda(z), \quad C_{44} = C_{55} = C_{66} = \mu(z) = \frac{E(z)}{2(1+\nu)}, \quad \lambda(z) = \frac{\nu E(z)}{(1-2\nu)(1+\nu)} \quad (12)$$

With $\lambda(z)$ and $\mu(z)$ are Lamé's constants.

The sandwich plate is subjected to a sinusoidal load in the following form.

$$q(x, y) = q_0 \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right) \text{ with } m = n = 1, \text{ and } q_{11} = q_0 \quad (13)$$

q_0 is the intensity of the load at the center of the rectangular sandwich plate.

The governing equations can be derived by applying the principle of virtual work, leading to the following:

$$\delta U + \delta K = 0 \quad (14)$$

The variation of strain energy δU of the plate is calculated by:

$$\delta U = \int_V \left[\sigma_x \delta \varepsilon_x + \sigma_y \delta \varepsilon_y + \sigma_z \delta \varepsilon_z + \tau_{xy} \delta \gamma_{xy} + \tau_{xz} \delta \gamma_{xz} + \tau_{yz} \delta \gamma_{yz} \right] dV \quad (15)$$

$$\delta U = \int_{-h/2}^{h/2} \int_{\Omega} \left[N_x \delta \varepsilon_x^0 + N_y \delta \varepsilon_y^0 + N_z \delta \varepsilon_z + N_{xy} \delta \varepsilon_{xy}^0 + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + S_{yz}^s \delta \gamma_{yz}^s + S_{xz}^s \delta \gamma_{xz}^s \right] d\Omega dz \quad (16)$$

The variation of the potential energy δK of the plate can be calculated as follows:

$$\delta K = - \int_{\Omega} q(x, y) \delta (w_b + w_s + w_{st}) d\Omega \quad (17)$$

$q(x, y)$: is the load applied transversely on the top surface of the FGM sandwich plate.

Where Ω is the top surface, the stress resultants N , M and S can be expressed as follows:

$$\begin{aligned} \begin{Bmatrix} N_x & N_y & N_{xy} \\ M_x^b & M_y^b & M_{xy}^b \\ M_x^s & M_y^s & M_{xy}^s \end{Bmatrix} &= \sum_{n=1}^3 \int_{h_n}^{h_{n+1}} (\sigma_x, \sigma_y, \tau_{xy})^{(n)} \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz, \\ N_z &= \sum_{n=1}^3 \int_{h_n}^{h_{n+1}} (\sigma_z)^{(n)} g'(z) dz, (S_{xz}^s, S_{yz}^s) = \sum_{n=1}^3 \int_{h_n}^{h_{n+1}} (\tau_{xz}, \tau_{yz})^{(n)} g(z) dz \end{aligned} \quad (18)$$

Substituting equation (10) into equation (18) and integrating across the thickness of the plate, the stress resultants are written as:

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ A & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ K^b \\ K^s \end{Bmatrix} + \begin{bmatrix} L \\ L^s \\ R \end{bmatrix} \varepsilon_z^0, \quad S = A^s \gamma, \quad N_z = R_s \varphi + L(\varepsilon_x^0 + \varepsilon_y^0) + L^s(k_x^b + k_y^b) + R(k_x^s + k_y^s) \quad (19)$$

$$N = \{N_x, N_y, N_{xy}\}^t, \quad M^b = \{M_x^b, M_y^b, M_{xy}^b\}^t, \quad M^s = \{M_x^s, M_y^s, M_{xy}^s\}^t \quad (20)$$

$$\varepsilon = \{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\}^t, \quad K^b = \{k_x^b, k_y^b, k_{xy}^b\}^t, \quad K^s = \{k_x^s, k_y^s, k_{xy}^s\}^t \quad (21)$$

$$\begin{aligned} A &= \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \\ B^s &= \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, \quad D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \quad H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix} \end{aligned} \quad (22)$$

$$S = \{S_{xz}^s, S_{yz}^s\}, \quad \gamma = \{\gamma_{xz}, \gamma_{yz}\}, \quad A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \quad (23)$$

$$\begin{Bmatrix} L \\ L^s \\ R \\ R^s \end{Bmatrix} = \sum_{n=1}^3 \int_{h_n}^{h_{n+1}} \lambda(z) \begin{Bmatrix} 1 \\ z \\ f(z) \\ g'(z) \frac{1-\nu}{\nu} \end{Bmatrix} g'(z) dz \quad (24)$$

Where A_{ij} , B_{ij} , D_{ij} ... ($i, j=1,2,6$) are the stiffness components and inertias defined by

$$(A_{ij}, B_{ij}, B_{ij}^s, D_{ij}, D_{ij}^s, H_{ij}^s) = \sum_{n=1}^3 \int_{h_n}^{h_{n+1}} \lambda(z) (1, z, z^2, f(z), z f(z), f^2(z)) \begin{Bmatrix} \frac{1-\nu}{\nu} \\ 1 \\ \frac{1-2\nu}{2\nu} \end{Bmatrix} \quad (25)$$

$$(A_{22}, B_{22}, B_{22}^s, D_{22}, D_{22}^s, H_{22}^s) = (A_{11}, B_{11}, B_{11}^s, D_{11}, D_{11}^s, H_{11}^s) \quad (26)$$

$$A_{ij}^s = \sum_{n=1}^3 \int_{h_n}^{h_{n+1}} \mu(z) [g(z)]^2 dz, \quad (i, j = 4, 5) \quad (27)$$

Substituting equations (18) to (20) and (25) into (15) then in equation (13), integrating by parts, and collecting the coefficients of $\delta u_0, \delta v_0, \delta w_b, \delta w_s, \delta \varphi$ the following equations of motion are obtained:

$$\begin{aligned}
 \delta u_0 : \quad & \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\
 \delta v_0 : \quad & \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\
 \delta w_b : \quad & \frac{\partial^2 M_x^b}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + q = 0 \\
 \delta w_s : \quad & \frac{\partial^2 M_x^s}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} + q = 0 \\
 \delta \varphi : \quad & \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} - N_z = 0
 \end{aligned} \tag{28}$$

II.4 NAVIER'S SOLUTION FOR RECTANGULAR SANDWICH PLATES WITH SIMPLE SUPPORT

In Navier's solution approach, the displacement fields are expressed as double Fourier series conforming the simply supported boundary conditions, as follows

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_b \\ w_s \\ \varphi \end{Bmatrix} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} \cos(\lambda x) \sin(\mu y) \\ V_{mn} \sin(\lambda x) \cos(\mu y) \\ W_{bmn} \sin(\lambda x) \sin(\mu y) \\ W_{smn} \sin(\lambda x) \sin(\mu y) \\ \phi_{mn} \sin(\lambda x) \sin(\mu y) \end{Bmatrix} \tag{29}$$

$U_{mn}, V_{mn}, W_{bmn}, W_{smn}, \phi_{mn}$ are arbitrary parameters to be determined. $\lambda = \frac{m\pi}{a}$ and $\mu = \frac{n\pi}{b}$ not to be confused with Lamé's constants $\lambda(z)$ and $\mu(z)$. After substitution one obtains the following equation in matrix form.

$$[K]\{\Delta\} = \{P\} \tag{30}$$

where $\{\Delta\} = \{U_{mn}, V_{mn}, W_{bmn}, W_{smn}, \phi_{mn}\}^t$ and $[K]$ is a symmetric matrix defined by

$$[K] = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} \\ k_{12} & k_{22} & k_{23} & k_{24} & k_{25} \\ k_{13} & k_{23} & k_{33} & k_{34} & k_{35} \\ k_{14} & k_{24} & k_{34} & k_{44} & k_{45} \\ k_{15} & k_{25} & k_{35} & k_{45} & k_{55} \end{bmatrix} \tag{31}$$

In which

$$\begin{aligned}
 k_{11} &= -(A_{11}\lambda^2 + A_{66}\mu^2) \\
 k_{12} &= -\lambda \mu (A_{12} + A_{66}) \\
 k_{13} &= \lambda [B_{11}\lambda^2 + (B_{12} + 2B_{66})\mu^2] \\
 k_{14} &= \lambda [B_{11}^s\lambda^2 + (B_{12}^s + 2B_{66}^s)\mu^2] \\
 k_{15} &= L\lambda \\
 k_{22} &= -(A_{66}\lambda^2 + A_{22}\mu^2) \\
 k_{23} &= \mu [(B_{12} + 2B_{66})\lambda^2 + B_{22}\mu^2] \\
 k_{24} &= \mu [(B_{12}^s + 2B_{66}^s)\lambda^2 + B_{22}^s\mu^2] \\
 k_{25} &= L\mu \\
 k_{33} &= -(D_{11}\lambda^4 + 2(D_{12} + 2D_{66})\lambda^2\mu^2 + D_{22}\mu^4) \\
 k_{34} &= -(D_{11}^s\lambda^4 + 2(D_{12}^s + 2D_{66}^s)\lambda^2\mu^2 + D_{22}^s\mu^4) \\
 k_{35} &= -L^s(\lambda^2 + \mu^2) \\
 k_{44} &= -(H_{11}^s\lambda^4 + 2(H_{11}^s + 2H_{66}^s)\lambda^2\mu^2 + H_{22}^s\mu^4 + A_{55}^s\lambda^2 + \mu^2) \\
 k_{45} &= -[A_{44}^s\lambda^2 + A_{55}^s\mu^2 + R(\lambda^2 + \mu^2)] \\
 k_{55} &= -(A_{44}^s\lambda^2 + A_{55}^s\mu^2 + R^s)
 \end{aligned} \tag{32}$$

The components of the generalized force vector $\{P\}^T = \{P_1, P_2, P_3, P_4, P_5\}$ are given by

$$\{P\}^T = \{0, 0, -q_0, -q_0, 0\} \quad (33)$$

Using the commercial software Abaqus, the user-defined material is introduced using the Fortran code UMAT [32] to perform the finite element analysis. The Eight-node brick element (C3D8) is used with a full integration (see Figure 3).

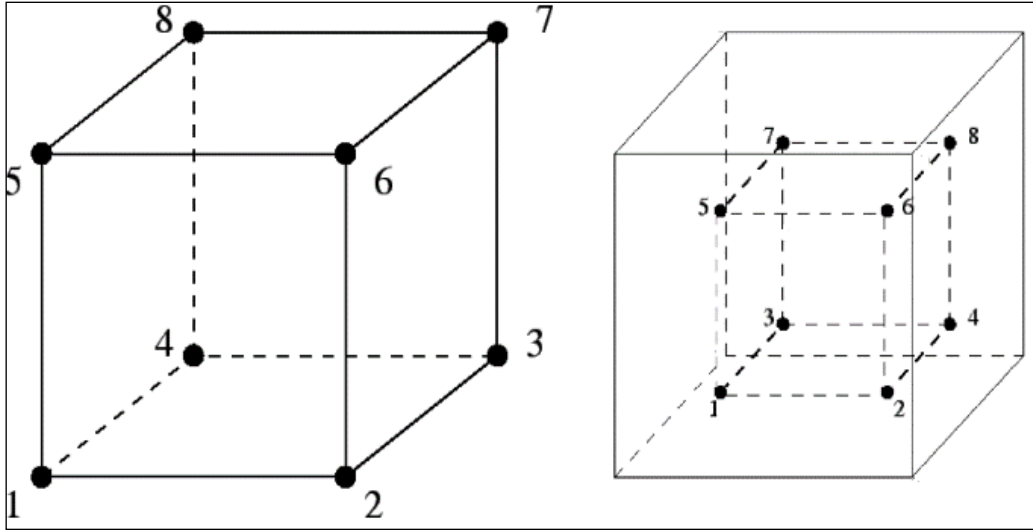


Figure 3: Nodes and integration points for C3D8 element.

Source: Authors, (2026).

Before using the UMAT a study of convergence and validation is performed. A convergence of three significant digits is adopted for all results presented.

III. RESULTS AND DISCUSSION

For the validation we consider a porous ($\alpha=0.1$) square sandwich plate with different layer thickness ratios (1-8-1) subjected to a sinusoidal load. The composition of the layers:

$$V_c = \begin{cases} ((z - h_1) / (h_2 - h_1))^k, h_1 \leq z \leq h_2 & \text{layer 1} \\ 1, h_2 \leq z \leq h_3 & \text{layer 2} \\ ((z - h_4) / (h_3 - h_4))^k, h_3 \leq z \leq h_4 & \text{layer 3} \end{cases} \quad (34)$$

$$\begin{aligned} E^{(1)}(z) &= (E_c - E_m) V^{(1)}(z) + E_m - \frac{\alpha}{2}(E_c + E_m) \\ E^{(2)}(z) &= (E_c - E_m) V^{(2)}(z) + E_m - \frac{\alpha}{2}(E_c + E_m) \\ E^{(3)}(z) &= (E_c - E_m) V^{(3)}(z) + E_m - \frac{\alpha}{2}(E_c + E_m) \end{aligned} \quad (35)$$

The following dimensionless forms are considered:

$$\hat{w} = \frac{10hE_0}{a^2q_0} w\left(\frac{a}{2}, \frac{b}{2}, 0\right), \bar{\sigma}_x = \frac{10h^2}{a^2q_0} \sigma_x\left(\frac{a}{2}, \frac{b}{2}, \frac{h}{2}\right), \bar{\sigma}_z = \frac{h}{aq_0} \sigma_z\left(\frac{a}{2}, \frac{b}{2}, \frac{h}{2}\right), \bar{\tau}_{xz} = \frac{h}{aq_0} \tau_{xz}\left(0, \frac{b}{2}, 0\right) \quad (36)$$

Geometric, materials, and load properties are: $E_c=380\text{GPa}$, $E_m=70\text{GPa}$, $\nu=0.3$ and $q_0=100\text{GPa}$.

Table 1 displays the validation of the analytical formulation and the FEM model. The obtained results are compared to [33]. [33] neglected the stress through the thickness. The results show good agreement for deflection and shear stress. As for the normal stress, a constant relation of 22/3 exists between the two sets of results.

Table 1: Non-dimensional deflection and stresses of a porous square sandwich plate for various volume fraction indices and side-to-thickness ratios.

k	Source	a/h	$\hat{w}$		$\bar{\tau}_{xz}$		$\bar{\sigma}_x$		$\bar{\sigma}_z$	
			$a/h=4$	$a/h=10$	$a/h=4$	$a/h=10$	$a/h=4$	$a/h=10$	$a/h=4$	$a/h=10$
0	Huang [33]	$\sigma_z=0$	0.0171	0.0842	0.2452	0.2462	2.0453	1.9487	-0.0000	-0.0000
	Present ψ_3		0.0169	0.0828	0.2451	0.2462	2.0986	1.9955		
	Present ψ_1	$\sigma_z \neq 0$	0.0164	0.0822	0.2420	0.2429	2.2232	2.0699	0.2314	0.2258
	Present ψ_2		0.0164	0.0823	0.2376	0.2386	2.1706	2.0062	0.1826	0.0728
	Present ψ_3		0.0164	0.0821	0.2444	0.2454	2.2673	2.1246	0.2740	0.3623
	Present FEM		0.0164	0.0817	0.2349	0.2342	2.0522	1.9488	0.1156	0.0313
1	Huang [33]	$\sigma_z=0$	0.0212	0.1080	0.2555	0.2563	2.6429	2.5209	-	-
	Present ψ_3		0.0210	0.1062	0.2555	0.2562	0.3603	0.3430	0.0000	0.0000
	Present ψ_1	$\sigma_z \neq 0$	0.0202	0.1054	0.2521	0.2527	0.3826	0.3577	0.0412	0.0434
	Present ψ_2		0.0202	0.1055	0.2472	0.2478	0.3722	0.3447	0.0320	0.0127
	Present ψ_3		0.0202	0.1053	0.2550	0.2556	0.3910	0.3683	0.0488	0.0692
	Present FEM		0.0202	0.1054	0.2527	0.2556	0.4005	0.3380	0.0726	-0.0126
3	Huang[33]	$\sigma_z=0$	0.0240	0.1243	0.2616	0.2621	3.0494	2.9124	-	-
	Present ψ_3		0.0237	0.1222	0.2616	0.2621	0.4157	0.3963	0.0000	0.0000
	Present ψ_1	$\sigma_z \neq 0$	0.0228	0.1213	0.2581	0.2585	0.4419	0.4144	0.0483	0.0531
	Present ψ_2		0.0228	0.1213	0.2529	0.2533	0.4290	0.3981	0.0373	0.0148
	Present ψ_3		0.0228	0.1211	0.2611	0.2616	0.4521	0.4274	0.0574	0.0840
	Present FEM		0.0229	0.1218	0.2647	0.2681	0.4046	0.2964	0.1257	0.0408
5	Huang[33]	$\sigma_z=0$	0.0250	0.1304	0.2636	0.2641	3.2019	3.0599	-	-
	Present ψ_3		0.0247	0.1282	0.2636	0.2641	0.4365	0.4163	0.0000	0.0000
	Present ψ_1	$\sigma_z \neq 0$	0.0238	0.1272	0.2601	0.2605	0.4641	0.4358	0.0510	0.0567
	Present ψ_2		0.0237	0.1273	0.2548	0.2552	0.4503	0.4183	0.0392	0.0155
	Present ψ_3		0.0238	0.1271	0.2632	0.2636	0.4750	0.4496	0.0605	0.0897
	Present FEM		0.0239	0.1280	0.2690	0.2726	0.5781	0.4004	0.2073	-0.0745

Source: Authors, (2026).

For the comparison, we consider a porous SFGM square sandwich plate. The three layers have equal thicknesses (1-1-1). The plate is simply supported. The upper surface of the sandwich plate is loaded with a sinusoidal force. Porosity distribution is given by equation (3) with $\alpha=0.1$. Notice that, due to this porosity the distribution of Young's modulus is not smoother (see Figure 9). Geometric, materials, and load properties are: $a/h=4$, $a=b$, $E_c=151\text{GPa}$, $E_m=70\text{GPa}$, $\nu=0.3$, $k=2$ and $q_0=100\text{GPa}$. From the displacement field of equation (4) we can see that the displacement is directly related to the shape functions. We select three different (see Figure 4) and widely used shape functions to study their influence on the through-thickness displacement and stress distributions. Figures 5-8 show the through-thickness displacement and stress distribution. The first comparison between curves ψ^{1s} (without the thickness effect) and ψ^1 (including the thickness effect) shows the big influence of the thickness effect term, especially on the displacement w and the σ_x stress. The influence of the different shape functions (in equations (4), (5) and (6)) exists for the displacement and the different stresses. For the stress the influence is considerable. Notice that a discontinuity in stress distributions is remarkable between the core and sheets of the sandwich plate due to the porosity term ($\alpha/2*(E_c+E_m)$). A comparison with finite element analysis is provided. Studies of convergence and validation are provided for every finite element result. The higher-order theory of shear deformation predicts a symmetric displacement distribution, although the top surface is more responsive to the applied load than the bottom surface. This phenomenon is well illustrated by the FEA results (see Figure 5). For stress distribution, the high-order shear deformation field provides a symmetric distribution of stress, even when the upper surface is loaded and the lower surface is not (see Figure 7). This phenomenon is also illustrated by the FEA results (see Figure 5 and Figure 7). To ensure, a second FEA (FEA2) is performed with half load applied on both upper and lower surfaces. The results are presented in Figure 10 and Figure 11.

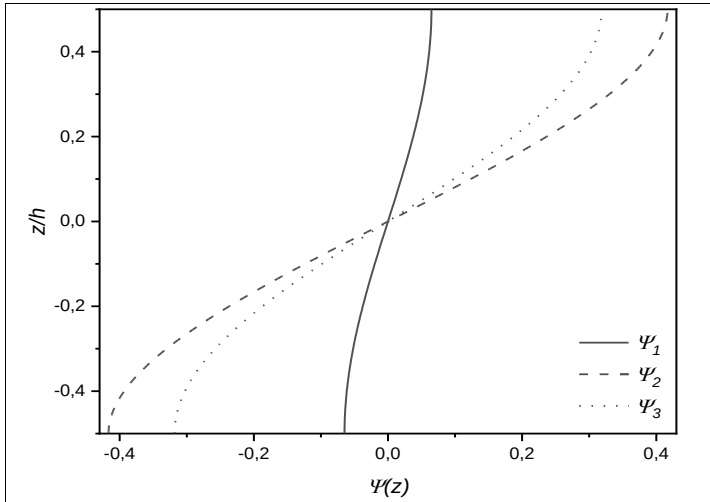


Figure 4: Shape functions plot.
Source: Authors, (2026).

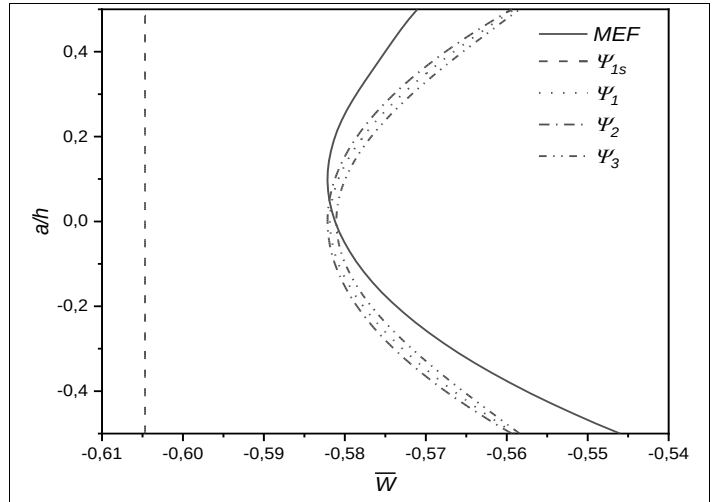


Figure 5: Through-thickness displacement distribution.
Source: Authors, (2026).

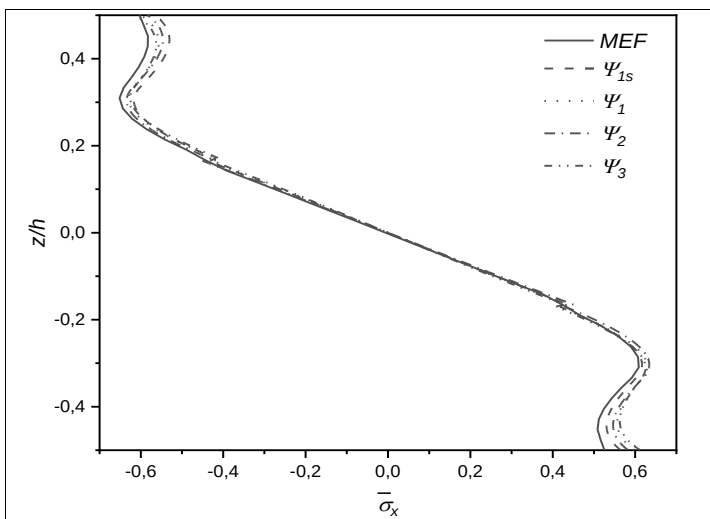


Figure 6: Through-thickness σ_x stress distribution.
Source: Authors, (2026).

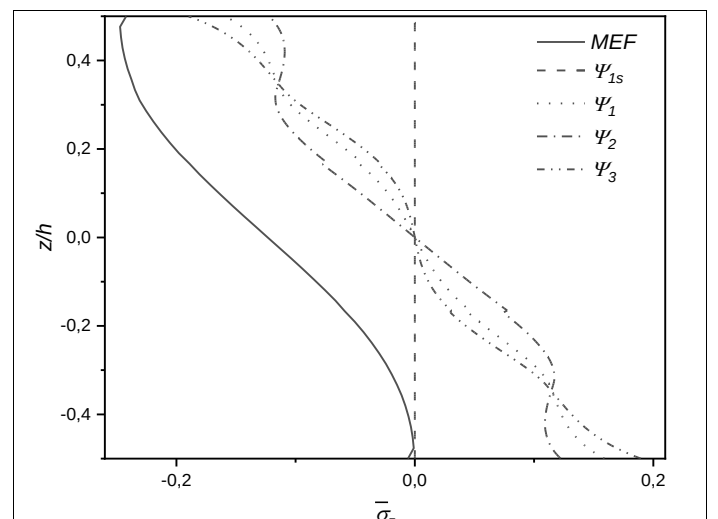


Figure 7: Through-thickness σ_z stress distribution.
Source: Authors, (2026).

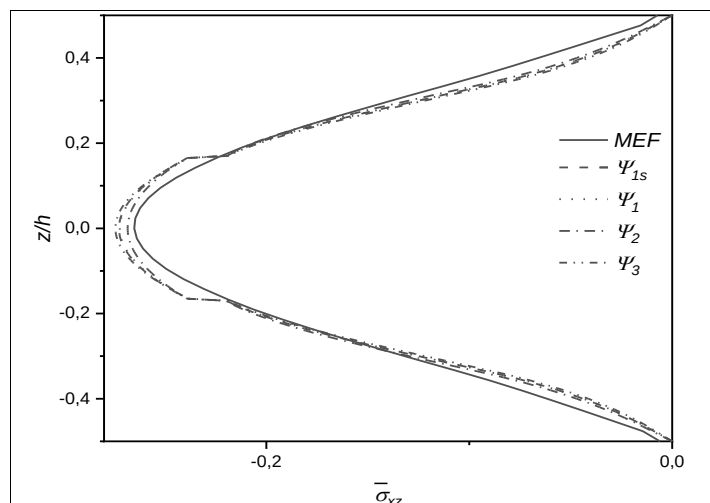


Figure 8: Through-thickness σ_{xz} stress distribution.
Source: Authors, (2026).

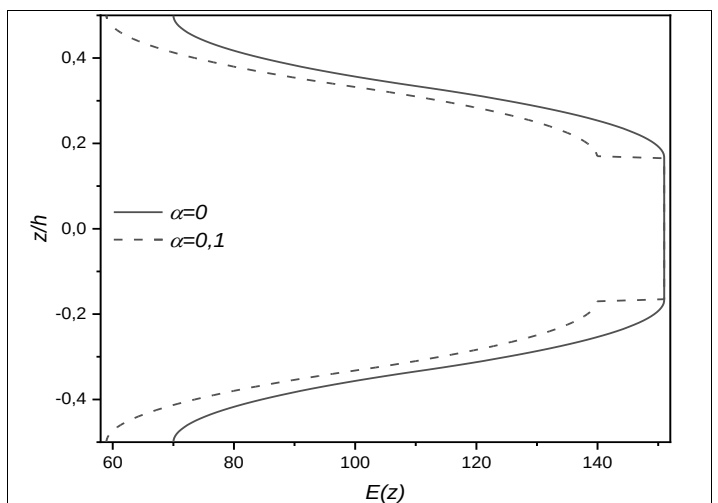


Figure 9: Through-thickness ($E(z)$) Young modulus distribution for perfect ant porous sandwich plate.
Source: Authors, (2026).

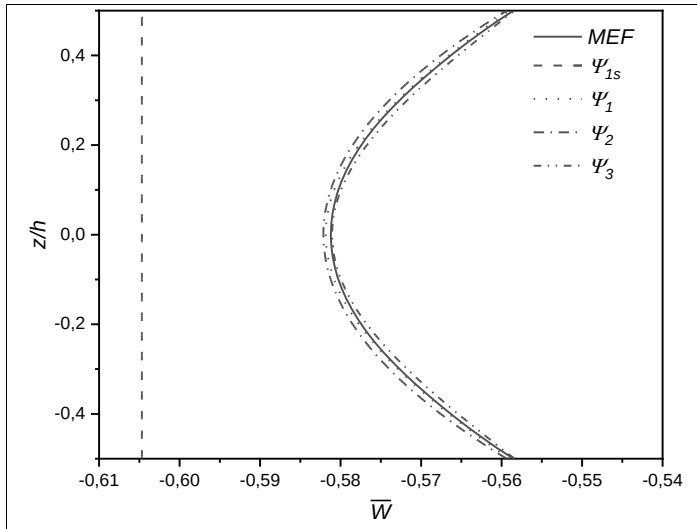


Figure 10: Through-thickness displacement distribution with the half load applies on the top and the bottom faces.
Source: Authors, (2026).

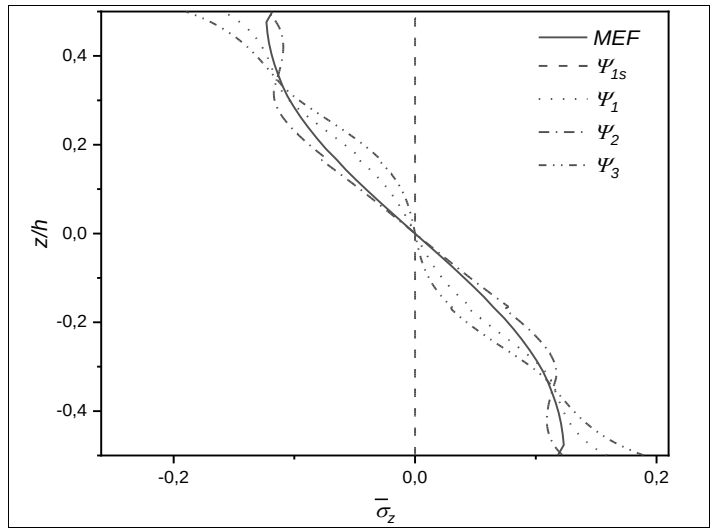


Figure 11: Through-thickness σ_z stress distribution with the half load applied on the top and the bottom faces.
Source: Authors, (2026).

IV. CONCLUSIONS

The thickness effect term exerts a substantial influence, especially on the displacement and the stress. The influence of the various shape functions is evident in both displacement and stresses. Notably, it has a significant impact on stress. The analytical formulation keeps giving symmetric distribution of displacement and stress even if it is not; when the load is apply on the upper surface of the plate. The porosity term causes a substantial stress discontinuity separating the core and the face sheets of the sandwich plate.

V. AUTHOR'S CONTRIBUTION

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Investigation: Youcef Beldjelili.

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