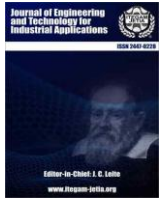




ISSN ONLINE: 2447-0228

ITEGAM-JETIA

Manaus, v.12 n.59, p.320-327. May/June, 2026.

DOI: <https://doi.org/10.5935/jetia.v12i59.3437>

RESEARCH ARTICLE

OPEN ACCESS

ENHANCED PITCH ANGLE OF AIRCRAFT SYSTEM USING FRACTIONAL MODEL REFERENCE ADAPTIVE CONTROL APPROACH

Benzenati Rahima*¹, Bensafia Yassine² and Lehouche Hocine³

¹Electrical Engineering Department, Faculty of Technology, Bejaia University 06000, Algeria.

²LISEA Laboratory, Electrical Engineering Department, Faculty of Applied sciences, University of Bouira 10000 ,Algeria.

³Industrial Technology and Information Laboratory (LTII), Faculty of Technology, Bejaia University 06000, Algeria.

¹<http://orcid.org/0009-0008-6545-6144>²<http://orcid.org/0000-0003-1760-3636>³<http://orcid.org/0000-0003-1392-6550>

Email: *rahima.benzenati@univ-bejaia.dz, y.bensafia@ univ-bouira.dz, hocine.lehouche@univ-bejaia.dz

ARTICLE INFO

Article History

Received: January 27, 2026

Reviewed: March 28, 2026

Accepted: April 28, 2026

Published: May 29, 2026

Keywords:

Fractional systems,
Aircraft System,
FMRAC approach,
Approximations methods.

ABSTRACT

Fractional calculus has been rediscovered by scientists and engineers in the last two decades and applied in an increasing number of fields, namely in the area of control theory. FMRAC (Fractional Model Reference Adaptive Control) is an advanced control strategy that extends traditional adaptive control techniques by incorporating fractional calculus. This approach is particularly beneficial in systems where dynamics are complex and exhibit non-integer order behavior. The current research work presents the use of the fractional Model Reference adaptive control (FMRAC) approach to improve the performances (rise time, setting time, overshoot and mean absolute error) for Aircraft by introducing fractional order integrator in the model reference system. To validate the arguments, effectiveness and performances analysis of the proposed FMRAC algorithm have been studied in comparison to the classical MRAC approach. Numerical simulation and analysis are presented to verify the best controller. The FMRAC approach gives the best results in terms of settling time, rise time, overshoot and mean absolute error. This approach can also be generalized to others fractional and integer systems in order to improve their performances and noise rejection.



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I. INTRODUCTION

Fractional-order (FO) differential calculus, or fractional calculus, is an area of mathematics concerned with the expansion of well-known differentiation and integration operations to arbitrary orders. Thanks to correspondence between two mathematicians, Leibniz and L'Hospital, science began to deal with the field about 1695. Even if the great popularity of fractional calculus is very recent mainly regarding its application in science and engineering, its history goes 300 years back. Particularly, control theory and applications is one of the major fields of application of fractional order systems, with a quickly growing quantity of theoretical and experimental research production [1], [2]. The reason for this success is due to the advantageous properties of fractional order control (FOC) systems and their interesting ability to improve the process robustness against disturbances and noises. A good confirmation is the fact that the first FOC scheme ever proposed in the literature, the so-called "Commande Robuste d'Ordre Non Entier" (CRONE) controller [3], deals with robust control. The applications of fractional order differentiation have attracted the attention of researchers from a wide variety of science disciplines especially from the fields of applied sciences [4], [5]. It is well-known that to address the parametric uncertainty in both linear and non-linear systems, adaptive control method is one of the best control techniques being used so far. However, the focus was on the use of integer order systems to implement adaptive control method [6-8]. In the domain of adaptive control, Model Reference Adaptive Control (MRAC) is a very attractive approach, because it offers a high level of performance while being very easy to implement. This scheme is mainly used to deal with unknown or slowly varying plants. The desired dynamical behavior of this adaptively controlled system is imposed by means of a reference model chosen by the designer. Adjustment of parameters is achieved using the error between the plant's output and the model reference output. Many control researchers are attempting to improve the MRAC robustness in order to deal efficiently in real time with the potential external disturbance that usually affects industrial plants.

Control engineering literature counts various robustification techniques that were introduced in elementary adaptive control algorithms [9]. Many efforts have been reported in the literature focusing on new MRAC control concepts design using fractional order differential operators, with important results for temporal behavior and robustness versus external perturbation [10-12]. Fractional Order Model Reference Adaptive Control (FOMRAC) became a leading research area in the field of automatic control engineering since its first apparition in the literature which was pioneered by the teams of vinagre et al [12] and bensafia et al. [13], [14]. The main contribution of this work is the use the FMRAC approach to improve the performances (rise time, setting time and overshoot) for Aircraft system by introducing fractional order integrator in the classical model reference system. The manuscript is organized as; firstly we have discussed the fundamentals of a fractional order system followed by the study mathematical model of the Aircraft system. Section three is an background on Model Reference Adaptive Control (MRAC) and MIT rule. Afterwards, the results obtained with performance analysis from simulation applying for Aircraft system using the integer and fractional MRAC approach is presented. Lastly, the conclusion along with future perspectives of the study is given.

II. OUSTALOUP APPROXIMATION METHOD

It is well-known that calculus is used to generalize the derivation or integration of various functions. However, A subfield of calculus is called fractional calculus, that normally used non-integer order for the generalization of derivatives or integrals of a function [15]. fractional approximation techniques are authorized to employ easily fractional order systems in many different application areas such as control theory [15-18], renewable energy [19], economical systems [20] etc. There are several characteristics that would decide the relative merits of any approximation such as differentiation order, frequency behavior or time responses etc. Several approximations are discussed here with respect to their comparative analysis with others [21], [22]. The available approximations belong to two different domains (frequency and time domains) which are specified as s-domain and z-domain, respectively. The Oustaloup method [23] is based on the function approximation from as;

$$G_f(s) = S^\alpha, \quad \alpha \in R^+ \quad (1)$$

By taking into account the rational function:

$$G_f(s) = K \prod_{k=1}^N \frac{s+w'_k}{s+w_k} \quad (2)$$

However, the poles, zeros, and gain can beevaluated as;

$$w'_k = w_b \cdot w_u^{(2k-1-\gamma)/N}, \quad w_k = w_b \cdot w_u^{(2k-1+\gamma)/N}, \quad K = w_h^\gamma$$

Where w_u represents the unity gain in frequency and the central frequency in a geometrically distributed frequency band. Let $w_u = \sqrt{w_h w_b}$, where, w_h and w_b represent the upper and lower frequencies, respectively. γ and N are the orders of derivative and filter, respectively.

III. MATHEMATICAL MODEL OF THE AIRCRAFT

An aircraft in flight can rotate about three distinct axes that converge at its center of gravity: yaw (rotation about the vertical axis), roll (rotation about the longitudinal axis), and pitch (rotation about the lateral axis).

Developing a comprehensive control model for aircraft systems presents significant complexity. To address this challenge, specific simplifying assumptions are adopted that enable focused analysis of pitch control dynamics exclusively.

The key assumptions include:

- **Force equilibrium:** During steady cruise flight at constant velocity and altitude, the aerodynamic forces (drag and lift) balance with thrust and weight in the respective horizontal and vertical planes.
- **Decoupled dynamics:** Pitch angle variations are assumed to produce no effect on aircraft velocity, thereby simplifying the analysis.

These assumptions result in the pitch control mathematical model presented in Figure 1 [24], which forms the basis for the subsequent control system design.

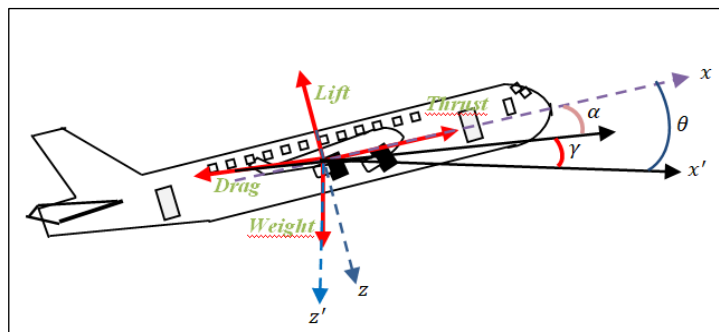


Figure 1: Pitch control description.
Source: Authors, (2026).

The dynamic response of an aircraft is described by the subsequent set of equations:

$$\frac{d\alpha}{dt} = \mu\Omega\sigma \left[-(C_L + C_D)\alpha + \frac{1}{\mu - C_L}q - (C_M \sin \gamma)\theta + C_L \right] \quad (3)$$

$$\frac{dq}{dt} = \frac{\mu\Omega}{2i_{yy}} [(C_m - \eta(C_L + C_D))\alpha + (C_m + \sigma(1 - \mu C_L))q + (\eta C_w \sin \gamma)\delta] \quad (4)$$

$$\frac{d\theta}{dt} = \Omega q \quad (5)$$

Where:

$\alpha, \theta, \gamma, \delta$: Attack, pitch, flight path and elevator deflection angles respectively.

C_T, C_D, C_L, C_W, C_M : Thrust, drag, lift, weight and pitch moment coefficients respectively.

σ, η : Constants

q : Pitch rate

ρ : Air density

S : Wing platform area

m : Mass of the aircraft

I_{yy} : Inertia normalized moment

Substituting the aircraft parameter values into Equations (3), (4), and (5) yields the following simplified modeling equations:

$$\frac{d\alpha}{dt} = -0.313\alpha + 56.7q + 0.232\delta \quad (6)$$

$$\frac{dq}{dt} = -0.0139\alpha - 0.426q + 0.0203\delta \quad (7)$$

$$\frac{d\theta}{dt} = 56.7q \quad (8)$$

The following is the Laplace transform of the preceding equations:

$$sA(s) = -0.313A(s) + 56.7Q(s) + 0.232\Delta(s) \quad (9)$$

$$sQ(s) = -0.0139A(s) - 0.426Q(s) + 0.0203\Delta(s) \quad (10)$$

$$s\Theta(s) = 56.7Q(s) \quad (11)$$

Finally, Aircraft pitch angle transfer function is given in Eq. (10).

$$G_p(s) = \frac{\Theta(s)}{\Delta(s)} = \frac{1.121s + 0.1774}{s^3 + 0.739s^2 + 0.9215s} \quad (12)$$

Where:

$\Delta(s)$: is the elevator deflection

$\Theta(s)$: is the pitch angle

The open loop step response is shown in Figure 2 with the input signal as $\delta = 0.2$. The open loop step response indicates that the system is unstable.

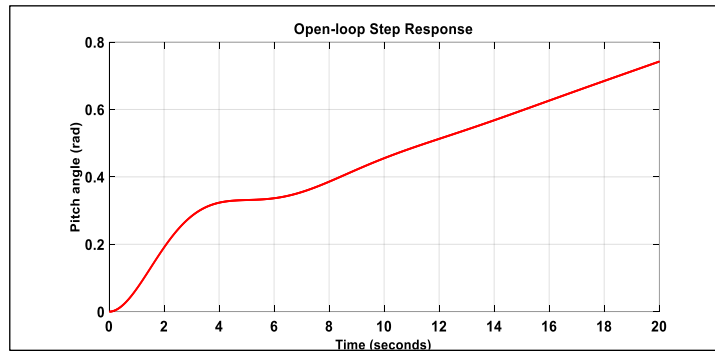


Figure 2: Open-loop step response of aircraft pitch angle.
Source: Authors, (2026).

A linearized model and simplified of aircraft system is considered in the form of a state-space system of equations [25]:

$$\dot{X}(t) = \begin{bmatrix} a_1 & a_2 \\ -1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} b \\ 0 \end{bmatrix} u(t) \quad (13)$$

$$y(t) = [1 \quad 0] x(t) \quad (14)$$

where:

- X is the state vector (e.g., angle of attack and pitch rate).
- U is the input (e.g., control surface actuation).
- Y is the output (e.g., angle of attack or altitude).
- $a_1 = -0.5$: Damping coefficient (aerodynamic stabilization effect).
- $a_2 = 1.2$: Aerodynamic gain (influence of angle of attack on dynamics).
- $b = 0.8$: Control gain (effectiveness of the control surface).

The system poles are determined by the eigenvalues of matrix A, which indicate the stability of the longitudinal dynamics. The vector B shows how the control input influences the aircraft dynamics and The vector C indicates which output is measured (here, probably the angle of attack).

IV. BACKGROUND ON MRAC: DESCRIPTION

The parallel structure (Figure 3) is the most famous MRAC structure, called the method of the output error; this approach involves plant model identification [13], [14] . Consider an single input, single output system, which can be a model of continuous time or discrete-time model:

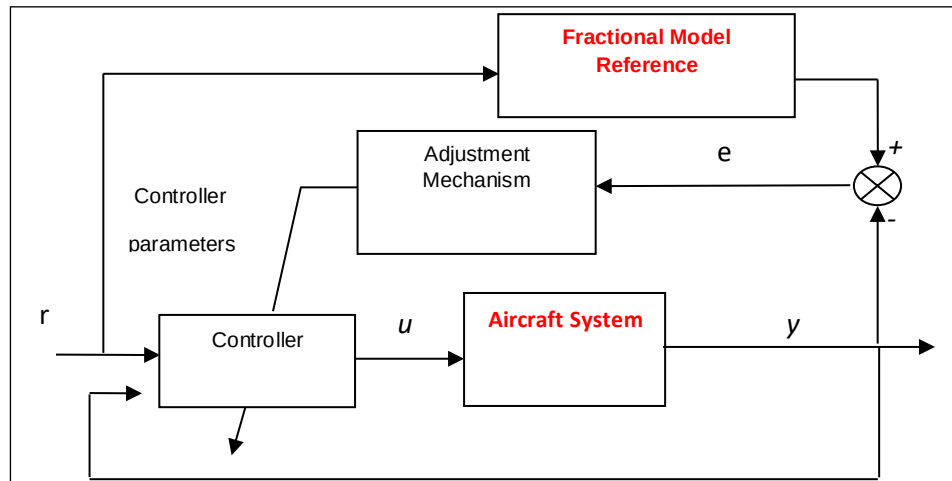


Figure 3: Fractional Model Reference Adaptive Control System.
Source: Authors, (2026).

Fractional Model Reference Adaptive Control (FMRAC) extends classical Model Reference Adaptive Control (MRAC) by incorporating fractional-order calculus into the control framework. This approach leverages fractional derivatives and integrals to achieve improved control performance, particularly for systems exhibiting memory effects and hereditary properties. Consider an input signal, the signal-output system, which can be a model of continuous time or discrete-time model:

$$y(t) = \frac{B}{A} u(t) \quad (14)$$

Where u is the control signal and y is the output signal. The symbols A and B denote polynomials in the differential operator p . It is assumed that the degree level $A \geq \deg(B)$, the system is causal [11]. We assume that we are trying to find a regulator that the relationship between the u_c and the desired signal output y_m , after having approximated the fractional reference model is given by:

$$y_m(t) = \frac{B_m}{A_m} u_c(t) \quad (15)$$

Where A_m and B_m are polynomials in the differential operator p . The generally linear control law is described as:

$$Ru = Tu_c - Sy \quad (16)$$

With R , S and T are polynomials. This control law is against a negative feedback with the transfer operator $-s/R$ and a direct reaction with the transfer operator $-T/R$. The gradient approach to MRAC is based on the assumption that the parameters change more slowly than the other variables in the system. Let e denote the error between the system output, y , and the reference output, y_m . Let θ denote the parameters to be updated. By using the criterion:

$$J(\theta) = \frac{1}{2} e^2 \quad (17)$$

The adjustment rule for changing the parameters in the direction of the negative gradient of J is that

$$\frac{d\theta}{dt} = -\gamma \cdot \frac{\partial J}{\partial \theta} = -\gamma \cdot e \cdot \frac{\partial e}{\partial \theta} \quad (18)$$

If it is assumed that the parameters change much more slowly than the other variables in the system, the derivative $\partial e / \partial \theta$, i.e., the sensitivity derivative of the system, can be evaluated under the assumption that θ is constant [11], [13]. The control law is given by the following equation:

$$u = -\theta^T (P_1 \varphi) \quad (19)$$

V. RESULTS AND DISCUSSIONS

The reference model defines the desired behavior that the aircraft must follow. It is designed to be stable, with a well-damped response to the reference input $r(t)$. The dynamics of the fractional reference model are described by:

$$D^\alpha x_m(t) = \begin{bmatrix} -1.5 & 0 \\ 0 & 0 \end{bmatrix} x_m(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(t) \quad (20)$$

$$y_m(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_m(t) \quad (21)$$

The error between the computed and measured output is used to determine the fitness function F that will be used for minimization. It is given by the following Mean Absolute Error (MAE) [26]:

$$MAE = \frac{\sum_{i=1}^N y(i) - y_m(i)}{N} \quad (22)$$

Where y is the measured output of Aircraft system and y_m is the desired output. Figure 4 shows the Pitch angle, control and Error signals of the Aircraft using the MRAC Controller

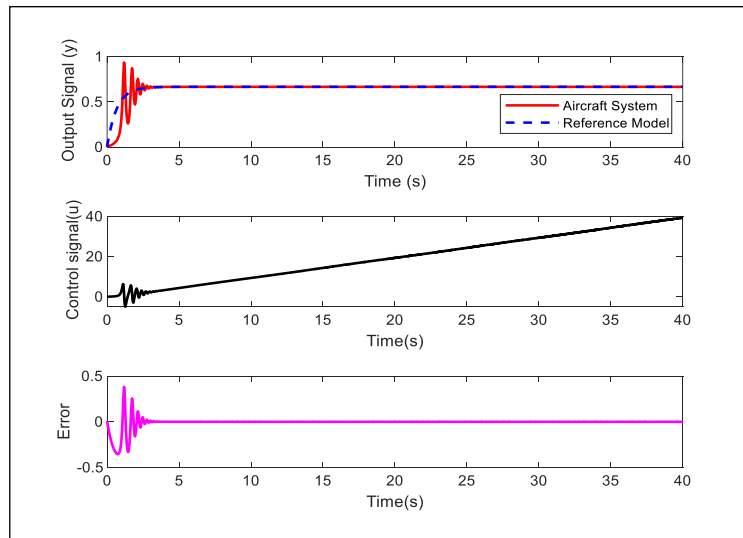


Figure 4: Pitch angle (y), control (u) and Error signal (e) of the Aircraft using the MRAC Controller
Source: Authors, (2026).

Figures 5 and 6 illustrate the pitch angle, control signal, and error response of the aircraft using the FMRAC controller with $\alpha = 0.1$ and $\alpha = 0.5$ respectively.

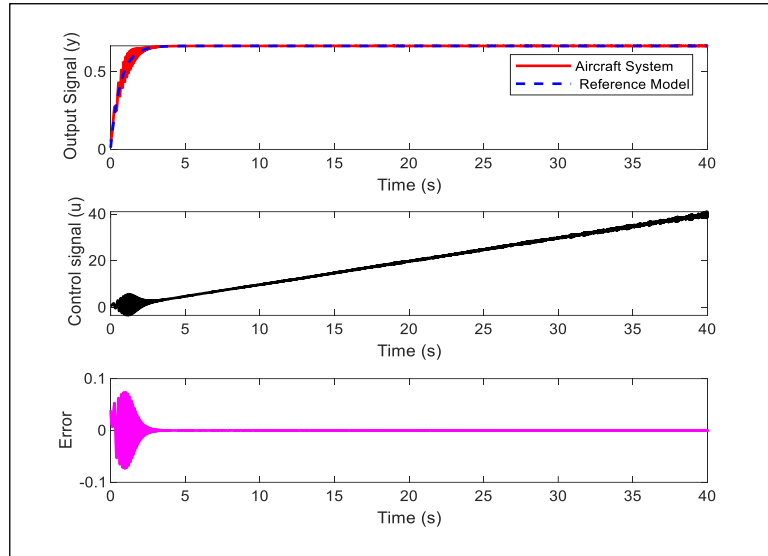


Figure 5: Pitch angle (y), control (u) and Error signal (e) of the Aircraft using the FMRAC Controller ($\alpha = 0.1$).
Source: Authors, (2026).

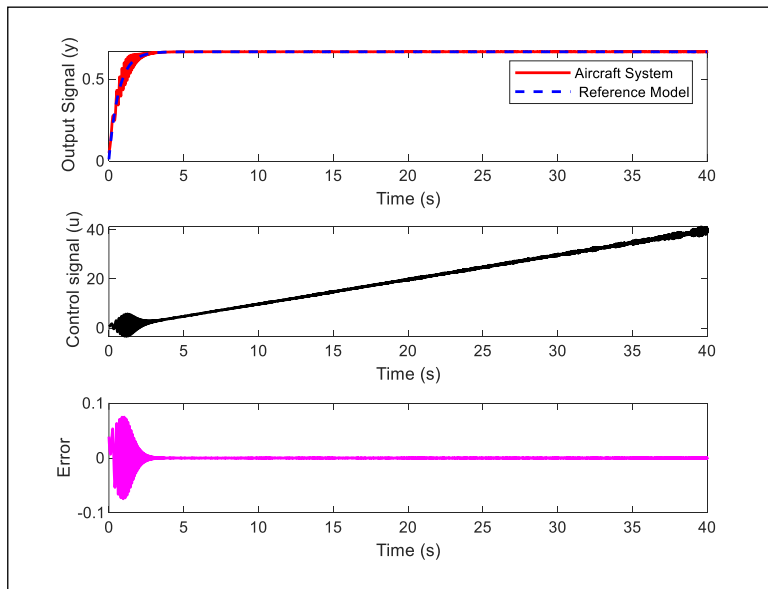


Figure 6: Pitch angle (y), control (u) and Error signal (e) of the Aircraft using the FMRAC Controller ($\alpha = 0.5$).
Source: Authors, (2026).

The performance analysis of the proposed FMRAC controller and the classical MRAC is given by the table1.

Table 1. Transient Response Stability Parameters of Aircraft System

Controllers	Overshoot [%]	Setting time [s]	Rise time [s]	Mean Absolute Error (Rad)
MRAC	0.26	2.6	1.03	0.01
FMRAC ($\alpha = 0.1$)	0	2.1	0.6	0.00022
FMRAC ($\alpha = 0.3$)	0	2.14	0.58	0.00021
FMRAC ($\alpha = 0.5$)	0	2.17	0.61	0.00030
FMRAC ($\alpha = 0.7$)	0	2.16	0.59	0.00025
FMRAC ($\alpha = 0.9$)	0	2.09	0.59	0.00024

Source: Authors, (2026).

We remark that the FMRAC Controller for Aircraft system give the good improvement of overshoot, setting time, rise time and mean absolute error comparatively to the Integer MRAC Controller results. It can be seen that the response of the proposed FMRAC controller is better than MRAC controller in terms of response time and transient behavior, as shown in Figure 2 and Figure 3. The effort required by the FOMRAC controller is smaller and less oscillatory than that of the MRAC controller. The great interest of the control engineering community for FOMRAC could be justified by the following four facts:

- Fractional order models are able to describe physical systems better than the classical 'integer-order' models.
- Introducing fractional order dynamics in the closed-loop control system, can improve the system's behaviour,
- The inherent properties of fractional-order systems, including a fairly wide gain margin and the memory effect, make the control system more robust against disturbances and noise than integer-order systems.
- FOMRAC configuration is still a very simple scheme and easy to implement compared to other adaptive controllers.

VI. CONCLUSIONS

In this paper we study the performance analysis using the MRAC Controller and the FMRAC Controller optimized by genetic algorithm applying to Aircraft system. The Fractional approach allows to improve the good performances of overshoot, setting time, rise time and mean absolute error comparatively to the Integer MRAC Controller results. The simulation studies show good performance of the proposed approach and confirm its superiority over Integer MRAC Controller. In future work, we will investigate the generalization of the fractional adaptive control approach to others fractional systems in order to improve their robustness and noise rejection.

VII. AUTHOR'S CONTRIBUTION

Conceptualization: Benzenati Rahima, Bensafia Yassine and Lehouche Hocine.

Methodology: Benzenati Rahima, Bensafia Yassine and Lehouche Hocine.

Investigation: Benzenati Rahima, Bensafia Yassine and Lehouche Hocine.

Discussion of results: Benzenati Rahima, Bensafia Yassine and Lehouche Hocine.

Writing – Original Draft: Bensafia Yassine .

Writing – Review and Editing: Benzenati Rahima, Bensafia Yassine and Lehouche Hocine.

Resources: Benzenati Rahima, Bensafia Yassine and Lehouche Hocine.

Supervision: Benzenati Rahima, Bensafia Yassine and Lehouche Hocine.

Approval of the final text: Benzenati Rahima, Bensafia Yassine and Lehouche Hocine.

VIII. REFERENCES

- [1] Monje.C. A., "Fractional-order systems and controls: fundamentals and applications," Springer, (2010).
- [2] K. Miller and B. Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, Wiley, New York, 1993.
- [3] M.Axtell, M. E. Bise, " In Fractional calculus application in control systems," IEEE Conference on Aerospace and Electronics, IEEE, pp 563-566, (1990).
- [4] Srinivasan .S, Karunanithi T., " Design of PI controller for bio-reactors for maximum production rate", International Journal of Chem-Tech Research, Vol.2, No.3, (2010), 1679-1685.
- [5] Wang .F. S, Juang. W. S, Chan. C. T, " Optimal tuning of PID controllers for single and cascade control loops", Chemical Engineering Communications, 132 , (1995),15-34.
- [6] Ladaci . S, Loiseau. J.J, Charef .A, " Adaptive Internal Model Control with fractional order Parameter ", Int. J. Adaptive Control and Signal Processing, nr 24,(2010), 944-960.
- [7] S. Ladaci and A. Charef, On Fractional Adaptive Control, Nonlinear Dynamics, vol.43, no.4, pp.365-378, 2006.
- [8] Bensafia, Y., Idir, A., Zemmit, A., & Khettab, K. " Performance Improvement of Aircraft pitch angle using the Fractional Order Adaptive PID Controller. Przegląd Elektrotechniczny, 99(5), (2023).
- [9] Z. Qu, J. F. Dorsey and D. M. Dawson, Model Reference Robust Control of a Class of SISO Systems, IEEE Trans. on Automatic Contr., vol.39, no.11, pp.2219-2234, 1994.
- [10] S Khelas, S Ladaci, Y Bensafia , 'Fractional order adaptive MRAC controller for an active suspension system ' , Algerian Journal Of Signals And Systems (Ajss), Vol.5, Issue 2, June, 2020.
- [11] Y. Bensafia and S. Ladaci, Adaptive control with fractional order reference model, Int. J. of Sciences and Techniques of Automatic control & computer engineering, vol.5, no.2, pp.1614-1623, 2011.
- [12] B. M. Vinagre, I. Petras, I. Podlubny and Y.-Q. Chen, Using Fractional Order Adjustment Rules and Fractional Order Reference Models in Model-Reference Adaptive Control, Nonlinear Dynamics, vol.29, pp.269-279, 2002.
- [13] Y Bensafia, K Khettab, S Ladaci, and A Chemori, 'Fractional Order Model Reference Adaptive Control for SCARA Robot Trajectory Tracking', International Journal of Industrial and Systems Engineering, 2018 .
- [14] Fang. Y, Xiaorong . H, and Tingting . T "Model reference adaptive control for fractional order systems with matched uncertainty", Proc. SPIE 12343, 2nd International Conference on Laser, Optics and Optoelectronic Technology (LOPET) , (2022)
- [15] Bensafia.Y, Idir.A, Khettab.K," Reducing the Delay Time and Tracking Trajectory of the Robot SCARA Using the Fractional PID Controller ", Science, Engineering and Technology, Vol. 5, No. 1, 2025.
- [16] Khettab . K , Ladaci . S, Bensafia . Y., "Fuzzy adaptive control of fractional order chaotic systems with unknown control gain sign using a fractional order Nussbaum gain ", In: IEEE/CAA Journal of Automatica Sinica, vol. 4(2), (2019), 1-8.
- [17] Oustaloup . A, Mathieu. B. , " La commande CRONE : du scalaire au multivariable ", HERMES, Paris,(1999).

- [18] Ynineb, A. R., Ladaci, S., “MRAC adaptive control design for an F15 aircraft pitch angular motion using Dynamics Inversion and fractional-order filtering”, *International Journal of Robotics and Control Systems*, 2(2): 240-250, (2022).
- [19] Neçaibia . A, Ladaci S., Charef . A, Loiseau. J.J, “Fractional order extremum seeking approach for maximum power point tracking of Photo-Voltaic panels”, *Frontiers in Energy* ; V 9, No.1, (2015), 43–53.
- [20] Dadras . S, Momeni H.R., “ Control of a fractional-order economical system ”, *Physica* 389,(2010),2434–2442.
- [21] ULLAH . N , WANG . S , KHATTAK .M. I “Fractional Order Fuzzy Backstepping Torque Control of Electrical Load Simulator”, *Przegląd Elektrotechniczny*, 89 (2013),nr. 5, 237-240.
- [22] Oustaloup. A, Sabatier . J, Lanusse .P, " From fractal robustness to CRONE control", *Fractionnal Calculus and applied Analysis*, (1999), 1-30.
- [23] Oustaloup . A, "La Dérivation Non Entière : Théorie, Synthèse et applications ", Hermès: Paris, (1995).
- [24] D. Izci, S. Ekinci, A. Demiroren, J. Hedley, HHO algorithm based PID controller design for aircraft pitch angle control system, in: 2020 Int. Congr. HumanComputer Interact. Optim. Robot. Appl, IEEE, 2020, pp. 1-6.
- [25] B. L. Stevens, F. L. Lewis, *Aircraft Control and Simulation*, Wiley, 2003.
- [26] Wang . W., Lu . Y, “ Analysis of the Mean Absolute Error (MAE) and the Root Mean Square Error (RMSE) in Assessing Rounding Model”, *IOP Conf. Series: Materials Science and Engineering* 324, (2018).