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OPTIMISED PLACEMENT OF MICRO-PMUS AND VOLTAGE ESTIMATION IN RADIAL DISTRIBUTION NETWORKS CONSIDERING ZERO INJECTION CONSTRAINTS

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ABSTRACT

This project presents a robust and efficient methodology for the optimal placement of Micro-Phasor Measurement Units (μ PMUs) in radial distribution networks, with particular attention to Zero Injection Bus (ZIB) constraints and high levels of Distributed Generation (DG). The proposed approach combines mathematical optimization and the Adaptive Mutated Practical Swarm Optimization (AMPSO) algorithm to perform accurate voltage estimation and enhance network observability. By strategically placing μ PMUs, the method reduces the number of measurement units needed while still meeting full observability and system monitoring goals. The framework is validated on the IEEE 33-bus test systems, showing improved performance, reduced computational complexity, and compatibility with renewable energy integration. Results highlight the project's effectiveness in supporting smart grid development through cost-efficient, scalable, and practical solutions.



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I. INTRODUCTION

The active distribution networks, denoted as ADN, have made power distribution networks on account of the fast use of renewable energy resources. This trend brings about new challenges for traditional state estimation (SE) techniques because increased complexity and variability of the network operations [1]. PMUs have recently emerged as an important technology in tackling such challenges with high-resolution real-time time data that improves the capabilities of monitoring and controlling the power grid [2]. The optimal place for the installation of the PMU in order to allow the total system Observability remains a vibrant and thriving area of research. There has been considerable research activity over the 20 years focused on the solution of the transmission network's Optimal PMU placement problem (OPP). However, more work is required in distribution networks. There are indications that Losses occur more in the distribution networks compared to the transmission networks. These include: networks, stressing the requirement for high-level control, monitoring, and reconfiguration of the above-mentioned networks [3], [4]. The use of μ PMUs is more economical for the smart grid, as it decreases the amount of smart meters that are needed [5]. Even traditional PMUs are quite expensive and not feasible for extensive distribution networks with multiple buses, μ PMUs provide alternatives because of their increased accuracy and sampling rate, which are necessary for the low values of X/R ratios of distribution networks [6]. Such precision is required in order to detect even more subtle variations in the angle and size of state variables. Nevertheless, much still needs to be done in the essential placement of μ PMUs in radial-type Distribution networks, especially about ZIBs. In [7], the researchers employed Integer Linear Programming (ILP) for resolving the OPP problem, taking into account the ZIB effect in the context of power transfer networks. Numerous meta-heuristic approaches have been recorded in [8].

These studies Specifically, it will highlight limitations such as ZIB existence and contingency. There will which tend to both exacerbate the nonlinearity and the scaling problem in scale distribution networks. In radial distribution networks, using dual employ line relays (DULR) and PMU for economic-based observability analysis [9] remains to be solved. Co-optimal PMU placement raises costs [10], whereas ILP-heuristic methods for distribution networks [11] do not take into account ZIB effects. Applications involving distribution level PMU (DPMU) [12] and exhaustive search algorithms of a transmission network [13] are also discussed. Full observability is ensured by the use of ILP frameworks [14], although complex radial ZIB configurations are still difficult. Some real-world cases of graph classification include. The limitations are taken into account in [15], including single PMU outages and line outages. The authors of [16] proposed a three-stage algorithm for PMU placement: guarantees full observation ability in transmission networks. However, this algorithm requires a pruning phase, which may add extra computation time for large distribution networks. A modified greedy algorithm for finding minimum spanning trees has. Algorithm is applied in [17] for resolving OPP problem in both normal and emergency situations, whereas [18] defines a multistage PMU placement. In the context of transmission networks, binary dragonfly algorithm outperforms both BPSO and modified BPSO.

Real-world limitations are taken care of by the such as ZIBs, integrated renewables, and topological changes [19-21]. In [22], a new ZIB modeling approach based on the virtual connection method is proposed. However, it may fail to produce the best results for total system observability. According to [23] It highlights the challenges-related to scalability in large radial networks and the complex nature of the ZIB model. Various studies have examined PMU placement and its challenges, this paper aims to bridge the gaps by presenting a novel approach for μ PMU placement and voltage estimation, specifically considering the complexities posed by ZIBs. This paper presents an optimized μ PMU placement strategy designed to ensure comprehensive observability and accurate voltage estimation in the presence of ZIBs. The study focuses on the economic placement of distribution-stage phasor measurements to enhance system observability, highlighting several key contributions: A technique to choose the best μ PMU locations for radial distribution systems while taking Zero injection constraints into account. A review of voltage estimation considering zero injections and μ PMU measurements. ZI buses decrease the number of μ PMUs needed, resulting in a fully observable and economically feasible network. The placement of μ PMUs is mostly determined by network topology and not affected by the installation of distributed generation units or other voltage- regulation apply [19-21].

II. PROBLEM STATEMENT

The following presents the placement of μ PMUs within radial distribution networks are critical in arriving at the exact voltage estimate that allows for efficient operation and monitoring of these networks. Due to the This is because of the difficulty involved with the measurement of voltage due to the complexity of these systems. It may not always be possible to maintain desired magnitudes and phase angles at all buses. Therefore, μ PMUs are placed accordingly to provide adequate observability, reducing the number of units required while still. This allows for an estimate of the voltage at each bus correctly. In radial distribution networks, ZIBs can be can be exploited in order to reduce the number of μ PMUs required. A ZIB is a bus where no active or reactive power is injected or consumed. Indirect state estimation is made possible by the information from ZIBs due to the fact that their voltages can be usually deduced from the neighbouring buses. This is further complicated by the inclusion of Photovoltaic (PV)-based DG yet contribute to this task by the added power injections, but these can also be measured and factored into the voltage estimation process. Therefore, the problem is to find the optimal location for placing μ PMUs in radial distribution networks with PV-based DG while-to consider it as a ZIB constrained problem, in order to minimize the number of μ PMUs and full observability of the network for the precise voltage estimation.

II.1 DECISION VARIABLES

Let;

N_b = Number of buses

N_{PMU} = Number of micro-PMUs

Binary placement vector:

$$\mathbf{z} = [z_1, z_2, \dots, z_{N_b}]$$

$$z_i = \begin{cases} 1, & \text{if micro-PMU is placed at bus } i \\ 0, & \text{otherwise} \end{cases}$$

State variables (voltage estimation):

$$\mathbf{x} = [V_1, \theta_1, V_2, \theta_2, \dots, V_{N_b}, \theta_{N_b}]^T$$

II.2 OBJECTIVE FUNCTION

Multi-objective optimisation (converted to single objective):

$$\min F = w_1 \sum_{i=1}^{N_b} z_i + w_2 \| \mathbf{z} - \mathbf{z}^{req} \| + w_3 \| \hat{\mathbf{x}} - \mathbf{x} \|_2^2$$

Where:

- First term → minimise number of micro-PMUs
- Second term → network observability
- Third term → voltage estimation error
- w_1, w_2, w^3 → weighting factors
- X → estimated voltage states

Measurement Model (With Micro-Pmus)

$$y = h(x) + e$$

Where:

- y → voltage & current phasor measurements
- $h(x)$ → nonlinear measurement function
- e → voltage estimation error

II.3 ZERO INJECTION BUS

For zero-injection buses

$$k \in \Omega_{ZIB}$$

Active power:

$$P_k = \sum_{j \in \mathcal{N}_k} V_k V_j (G_{kj} \cos \theta_{kj} + B_{kj} \sin \theta_{kj}) = 0$$

Reactive power:

$$Q_k = \sum_{j \in \mathcal{N}_k} V_k V_j (G_{kj} \sin \theta_{kj} - B_{kj} \cos \theta_{kj}) = 0$$

These constraints enhance state observability without additional PMUs.

II.4 AMPSO PARTICLE REPRESENTATION

Each particle represents

$$\mathbf{X}_i = [\mathbf{z}_i \quad \mathbf{x}_i]$$

Velocity update:

$$\mathbf{V}_i^{t+1} = \omega(t) \mathbf{V}_i^t + c_1(t) r_1 (\mathbf{P}_i - \mathbf{X}_i^t) + c_2(t) r_2 (\mathbf{G} - \mathbf{X}_i^t)$$

Position update:

$$\mathbf{X}_i^{t+1} = \mathbf{X}_i^t + \mathbf{V}_i^{t+1}$$

Binary update for PMU placement (sigmoid):

$$z_i^{t+1} = \begin{cases} 1, & \text{if } \text{sigmoid}(v_i^{t+1}) > \text{rand}() \\ 0, & \text{otherwise} \end{cases}$$

II.5 ADAPTIVE PARAMETER CONTROL

Adaptive inertia weight:

$$\omega(t) = \omega_{\max} - \left(\frac{\omega_{\max} - \omega_{\min}}{T} \right) t$$

II.6 MUTATION STRATEGY (AMPSO ENHANCEMENT)

If stagnation is detected:

$$\mathbf{X}_i^{mut} = \mathbf{X}_i + \sigma(t)\mathcal{N}(0,1)$$

Adaptive mutation strength:

$$\sigma(t) = \sigma_{\max} \left(1 - \frac{t}{T}\right)$$

II.7 CONSTRAINT HANDLING

Voltage limits:

$$V_i^{\min} \leq V_i \leq V_i^{\max}$$

PMU count constraint:

$$\sum_{i=1}^{N_b} z_i \leq N_{PMU}^{\max}$$

Penalty function:

$$F_{pen} = F + \lambda \sum \text{constraint violations}$$

Termination Condition

$$t = T \text{ or } |F^t - F^{t-1}| < \varepsilon$$

III. METHODOLOGY

The methodology for optimal placement of μ PMUs and DGs in radial distribution networks with ZIB constraints involves several key steps. First, system data, including bus and branch details, as well as DG characteristics, are collected. ZIBs are identified to leverage their properties for minimizing the number of μ PMUs needed for network observability. The placement problem is formulated as a MILP optimization, with the objective of determining the fewest number of μ PMUs required while ensuring full observability, accounting for both ZIB constraints and DG contributions. After solving the MILP, the optimal μ PMU locations are identified, and the voltage estimation process begins by initializing parameters and incorporating ZIB and DG constraints into the Jacobian matrix. An iterative AMPSO algorithm is applied to estimate the voltage profile across all buses, with the Jacobian matrix and measurements being updated at each step.

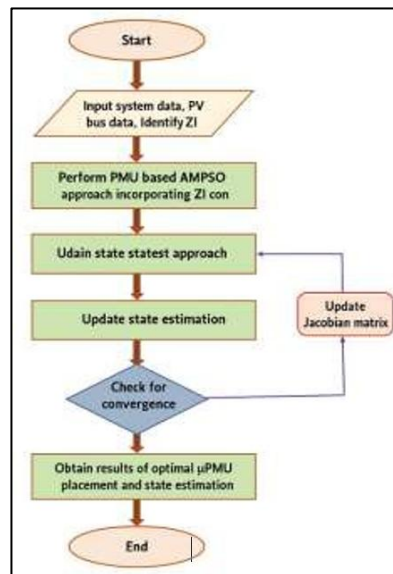


Figure 1: Flowchart: Process of optimal μ PMU placement and voltage estimation.
Source: Authors, (2026).

Steps are repeated until the convergence criteria are satisfied, ensuring the accuracy of voltage estimation. Once the final state estimation is obtained, the voltage profile is computed on the basis of μ PMU data, and this is verified for both observability and accuracy. Lastly, the final bus voltage estimates and optimal μ PMU installation locations are shown to show how ZIBs and DGs enhance the efficiency of μ PMU deployment and accurate voltage estimation in the network. Fig 1 shows the flowchart of optimal μ PMU placement and voltage estimation of active distribution network.

IV. SIMULATION RESULT AND DISCUSSION

This section details the outcomes of optimal μ PMU placement and examines the voltage profile across three scenarios. Simulations are carried out on the MATLAB platform. Three scenarios are used to test the suggested approach. (I) Condition I-Under normal conditions, (II) Condition II- consideration of ZI bus (III) the presence of Distributed Generators (DGs) in distribution networks.

IV.1 MICRO PMU PLACEMENT

IV.1.1 Materials and Methods

The following illustrates the placement of μ PMUs in radial distribution networks. play a crucial role in determining the precise voltage level that ensures efficient operation and monitoring of these networks. Because of the This is due to the difficulty associated with the measurement of voltage for these complex systems.

Table 1: Locations of micro-PMU in normal condition.

Feeder	Positions	No. Of Micro PMUS
33	2,5,8,11,14,17,20,24,29,30,32	11
69	2,4,6,9,14,17,20,23,26,28,31,34,38,40,44,47,50,51,55,57,59,62,65,68	24

Source: Authors, (2026).

Sometimes, it may be difficult to maintain the desired magnitude and phase value at all buses. Hence, the installation of μ PMUs follows in a manner as to supply the observability, minimizing units needed while still With this, an estimation of the voltage can be made at every bus. correctly. In radial distribution networks, ZIBs can be used can be leveraged in order to minimize the number of μ PMUs needed. A ZIB is a "bus where neither active nor reactive power is injected or consumed." Indirect state estimation is feasible through the information provided by ZIBs. because it is possible to generally conclude their voltages from the surrounding buses. This scenario gets further complicated by the consideration of Photovoltaic (PV)-based DG. but contribute to this process through the increased power injections, but these can also be measured and quantified in the estimate for the voltage process .Hence, the problem is as follows: To find the best location of placing μ PMUs in radial distribution networks, even where PV-based DG units, comprise to consider it as ZIB constrained, in order to keep the number of μ PMU as low as possible and full observability of the network for the accurate voltage It is an estimation.

IV.1.2 Condition Ii -Consideration of Zi Buses

ZI buses are points where there are neither load buses nor generator buses, which in many cases are found in radial distribution systems. The placement of μ PMUs at critical points and taking into consideration the ZI buses may lead to a reduction in the total μ PMUs required due to less impact of ZI buses on network observability. Taking for example if the ZI buses are considered and included in the IEEE 33-bus system, then changes may occur in the original placement plan for 11 PMUs, which may reduce in number for improved coverage. The selection for placement would include best strategic nodes that enable comprehensive coverage for the network while taking into consideration the role of ZI buses to optimize resource use and improve monitoring efficiency. In IEEE 69-bus systems, the same approach is considered for placement. Instead of using 24 PMUs for comprehensive coverage, incorporating ZI buses may optimize placement plans. By considering ZI buses for placement for μ PMUs, power companies may optimize resource monitoring investment and improve overall operational efficiency for power distribution networks. Table II illustrates points for ZI buses within a distribution network. ZIBs are points where either loads or generators are connected; they are usually best points for monitoring in radial power distribution networks placed for monitoring power analytics at critical points for power flow and system dynamics within power transmission networks. Table III conditions. This instance aptly demonstrates the benefits of ZIBs' incorporation within the placement strategy of μ PMUs. It helps in optimizing the usage of resources while still being able to make monitoring possible. It becomes possible to have equal network observability with reduced monitoring devices due to incorporation of ZIBs within the placement strategy of μ PMUs.

Table 2: Location of ZIB for test systems.

Systems	No. Of ZI Busses	ZI Bus Location
33	2	6,27
69	20	2,3,4,5,15,19,23,25, 30,31,32,38,42,44, 47,56,57,58,60,63

Source: Authors, (2026).

Table 3: Micro PMU position with and without ZIB.

Systems	No. of μ PMUs and position without ZIB	No. of μ PMUs and position with ZIB
33	11 & (2,5,8,11,14,17,20,24,27,30,32)	10&(2,5,8,11,14, 17,21,,29,32)
69	24&(2,6,9,14,17,20, 23,26,29,32,35,37,40, 43,46,47,50,52,55,58, 61,64,66,68) 20	16&(3,4,6,14,17,20, 26,34,41,45,59,51,54. 64,66,68)

Source: Authors, (2026).

When a PV-based DG is connected to the power grid, it injects main sufficient amounts of power supply current, phase angles, and frequency that is related to DG functioning. This direct measurement capability allows the system operator to measure the effect of DG on the power system instantly. Table IV shows the location of DGs and μ PMUs.

IV.1.3 Voltage Profile Estimation using Mpmus Technology with and without Zibs

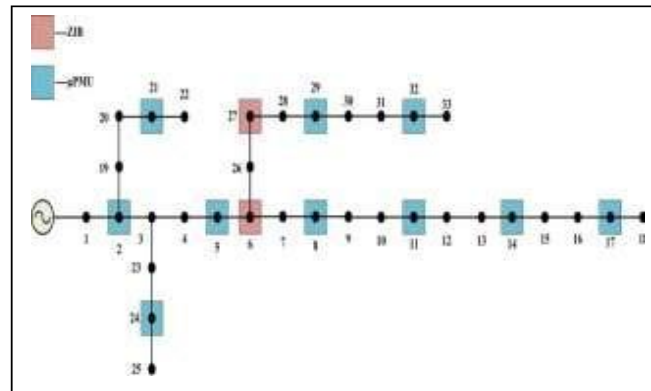
Table 4: Voltage Profile estimation using Micro PMUs with & without ZIBS.

System	DG buses	No. of μ PMUs	Positions
33	8,17,30	10	2,5,8,11,14,17,21,24,29,32
69	23,40,52,66	24	2,6,9,14,17,20,23,26,29,32,35,37,40,43,46,48,50,52,58,61, 64,66,68

Source: Authors, (2026).

IV.1.4 Results for the 33-Bussystem

Figure 2 and Figure 3 depict the 33 bus radial distribution network with μ PMUs, taking into account and disregarding ZIBs. In Figure 2, it is evident that another advantage, in this case, of including ZIBs, is minimizing the need for an effective number of μ PMUs to ensure overall observability. With carefully located μ PMUs and ZIBs factored into account, fewer units are needed to achieve equal levels of monitoring. In Figure 3, further evidence shows that voltage profile estimation is nearly equal with and without ZIBs, thus proving that ZIBs do not impair voltage estimation capability. Such optimization can result in the reduction of costs and a more optimized deployment of μ PMUs, thereby optimizing the deployment efficiency. Thus, using ZIBs can enhance the deployment efficiency of μ PMUs without affecting the voltage profile of the distribution system.

Figure 2: IEEE33 bus network with μ PMU and ZI.

Source: Authors, (2026).

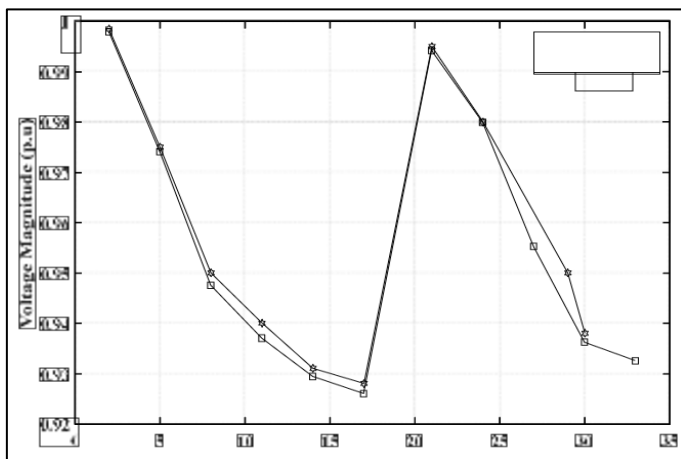


Figure 3: IEEE33bus voltage estimation with and without ZIBs.

Source: Authors, (2026).

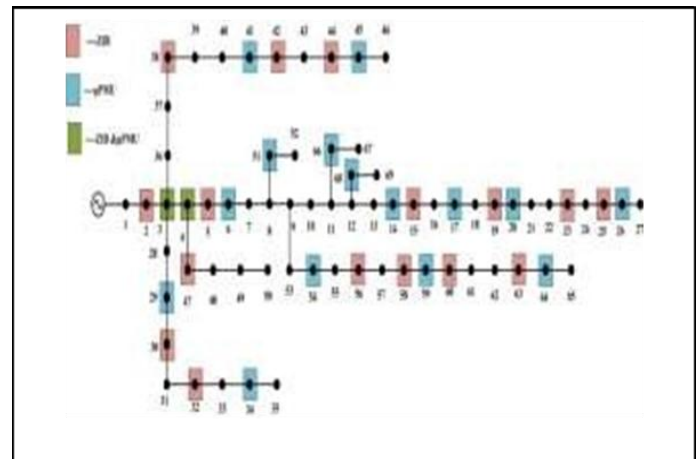


Figure 4: IEEE69 bus voltage estimation with and without ZIBs.

Source: Authors, (2026).

IV.1.5 Results for the 69-BusSystem

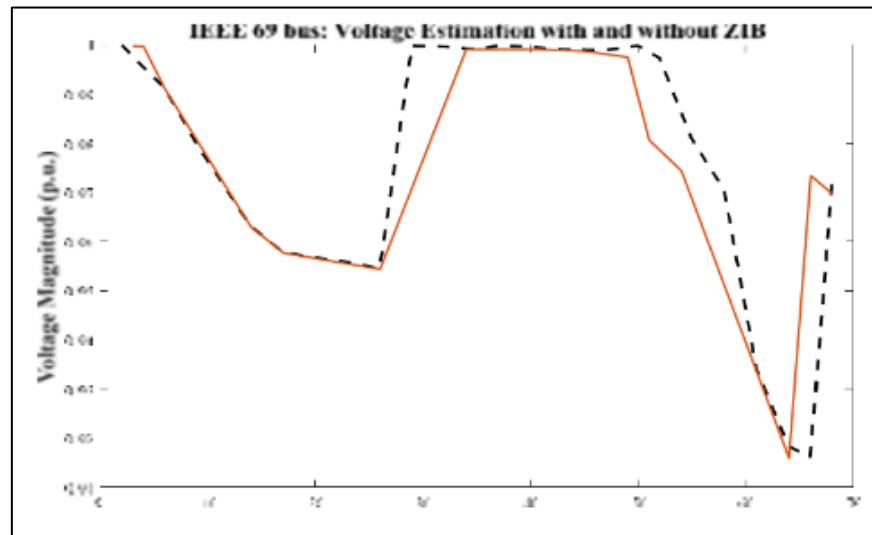


Figure 5: Results for the 69 Bus System.

Source: Authors, (2026).

Figure 4 highlights that incorporating ZIBs into the network configuration effectively reduces the number of μ PMUs required to maintain full network observability. Additionally, Figure 5 indicates that the voltage profile of the system remains largely unchanged with the inclusion of ZIBs. This demonstrates that utilizing ZIBs does not significantly impact the voltage stability or distribution within the network. Therefore, the integration of ZIBs can enhance the efficiency of μ PMU deployment, optimizing resource usage while maintaining a stable voltage profile across the distribution system.

Table 5: Comparative analysis under condition- II.

Reference	Observability	N μ PMU(IEEE 33)	N μ PMU(IEEE 69)
[15]	Yes	NR	18
[2]	No	NR	17
[8]	Yes	NR	18
Present work	Yes	10	16

Source: Authors, (2026).

Figure 4 and Figure 5 illustrate the 69-bus radial distribution network with μ PMUs installed at specific nodes, both considering and excluding ZIBs.

Table 6: Comparative analysis under condition-I.

Reference	Method	N μ PMU ^a (IEEE33)	N μ PMU ^a (IEEE 69)	Voltage estimation
[14]	Graphtheory	11	24	No
[15]	ILP	NR	24	No
[17]	ILP	11	NR ^b	No
[2]	ILP	NR	24	No
Present work	AMPSO	10	24	Yes

Source: Authors, (2026).

Figure 4 highlights that incorporating ZIBs into the network configuration effectively reduces the number of μ PMUs required to maintain full network observability. Additionally, Figure 5 indicates that the voltage profile of the system remains largely unchanged with the inclusion of ZIBs. This demonstrates that utilizing ZIBs does not significantly impact the voltage stability or distribution within the network. Therefore, the integration of ZIBs can enhance the efficiency of μ PMU deployment, optimizing resource usage while maintaining a stable voltage profile across the distribution system. Comparing the proposed method with the available literature shows the advantages of the latter in terms of μ PMU deployment and voltage estimation. It may be seen from Table V, Condition I that while methods such as graph theory and ILP require 10 μ PMUs for the IEEE 33-bus system and 24 μ PMUs for the IEEE 69-bus system, neither of them offers voltage estimation. The same number of μ PMUs are offered by the proposed AMPSO-based method but with additional voltage estimation capabilities that make it more useful. It follows from Table VI, Condition II that 10 μ PMUs for the IEEE 33-bus system and 16 for the IEEE 69-bus system are sufficient for the proposed approach against 17–18 μ PMUs required by other approaches that ensure full observability. It may, thus, be concluded that the proposed method is efficient for μ PMU placement and superior in terms of functionality since both reduction of the μ PMU requirement along with enhanced monitoring functionalities are possible.

V. CONCLUSIONS

The present study seeks to reduce the computational time required for the determination of optimal placement for μ PMUs under different configurations of distribution networks. It, therefore, introduces two key strategies toward the realization of this objective. First, it considers the identification of the optimum placement for μ PMUs in distribution networks, with and without ZIBs-nodes at which neither a load nor a generator is connected. Secondly, it evaluates the voltage profile of a radial distribution feeder using μ PMU technology in conjunction with the AMPSO algorithm in these network conditions. In this regard, the study proposes MILP to arrive at the optimum number and location μ PMUs that are necessary for normal network operation. Towards this end, the initial installation of μ PMUs takes place at critical points in the network. Subsequent installations take into account ZI buses to reduce the total number of μ PMUs required.

Comparative analyses with other methods demonstrate that the proposed strategy achieves better performance, minimizing the number of μ PMUs for both the IEEE 33-bus and IEEE 69-bus systems, while estimating the voltages-a feature lacking in most of the existing approaches. These results further demonstrate the practical benefits of incorporating ZIBs in μ PMU placement and provide a cost-effective scalable solution for enhanced monitoring in smart distribution networks. Future work can continue this approach for larger and more complex systems and investigate the possibility of real-time implementation.

VI. AUTHOR'S CONTRIBUTION

Conceptualization: Palepu Suresh Babu and Gavireddy Vamsidhar Reddy.

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Discussion of results: Gavireddy Vamsidhar Reddy, K. Venkata Ramana, T.Sravankumar, M Vinay Kumar.

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Supervision: Gavireddy Vamsidhar Reddy and K. Venkata Ramana.

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