



IMPROVED DC MOTOR SPEED REGULATION AND DISTURBANCE REJECTION VIA FRACTIONALIZED PID CONTROL USING THE OPTIMAL FRACTIONAL-ORDER SYSTEM APPROXIMATION METHOD

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ABSTRACT

In recent years, research has increasingly concentrated on fractional order systems and their approximation techniques. These systems have proven valuable for improving plant dynamics in both time and frequency domains. This paper introduces a novel approach for evaluating different approximation methods of fractional order systems and examining disturbance rejection in PID control of DC motors. The methodology involves fractionalizing an integer order derivative operator within the original integer system. The fractionalized terms are implemented using well-established approximation methods, and to identify the optimal technique, the responses of the original integer system are compared with those of the fractionalized systems. Simulation examples demonstrate that the fractionalization approach effectively identifies the most suitable method, serves as an effective comparative tool for different approximation techniques, and achieves strong disturbance rejection in PID control of DC motors. This approach is generalizable to other numerical approximation methods and applicable to broader control systems applications.



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I. INTRODUCTION

While fractional calculus has existed for over three centuries, its current widespread use stems primarily from its practical applications across scientific and engineering domains [1],[2]. The control theory field and fractional-order systems research have experienced particularly rapid advancement in both theoretical frameworks and experimental investigations [3]. Fractional differentiators and integrators are essential components in fractional feedback control systems and filter-based signal processing for controlling chaotic and complex dynamic systems [4],[5]. Within the s-domain, a fractional-order system is characterized as an infinite-dimensional filter featuring an irrational continuous-time transfer function [6].

The capabilities offered by fractional-order differentiation have attracted significant attention from researchers spanning numerous scientific disciplines, particularly within applied sciences [7]. For practical implementation of fractional-order systems across diverse application areas, several approximation techniques are widely utilized, including those developed by Oustaloup, Charef, and Matsuda [8]. This paper aims to introduce a new methodology for evaluating and comparing various approximation methods of fractional-order systems. The approach involves incorporating fractional-order filters into the classical design of the original system while preserving the general form of the equivalent transfer function. Consider an integrator element with transfer function $G(s)=(1/s)$ from the original integer-order system. The objective is to "fractionalize" this element in the following manner:

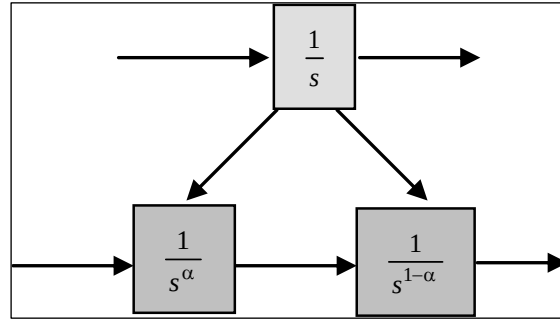


Figure 1: Integral operator fractionalization.
Source: Authors, (2026).

$$G(s) = G_{\alpha}(s) \cdot G_{1-\alpha}(s) = \left(\frac{1}{s^{\alpha}}\right) \cdot \left(\frac{1}{s^{1-\alpha}}\right) \quad (1)$$

where α : real number ($0 < \alpha < 1$).

The aim is not to modify the original overall system, but rather to utilize it as an innovative comparative tool for evaluating different approximation methods of fractional-order systems, given their advantageous properties. While this concept may seem relatively simple, it is anticipated to offer a valuable resource for design engineers working with industrial plant systems in real-world applications.

The paper is organized as follows: Section 2 presents the fundamental definitions of fractional-order systems. Section 3 describes various approximation methods for fractional-order transfer functions. Section 4 introduces the proposed fractionalized systems, including integrator and derivative operators, fractionalized first-order systems, and fractionalized second-order systems. Section 5 presents simulation results comparing three approximation methods (Oustaloup, Matsuda, and Charef) using both original integer-order systems and fractionalized systems. Finally, Section 6 provides concluding remarks and outlines future research directions.

II. FRACTIONAL ORDER SYSTEMS

The growing use of fractional calculus in practical applications is attributed to its superior capability in modeling physical systems compared to traditional integer-order approaches. While the fractional characteristics of most systems may be subtle, they remain present and meaningful [9,10]. Historically, integer-order models have been favored primarily due to the mathematical complexity involved in solving fractional differential equations. Nevertheless, recent research has established the efficacy of fractional calculus across numerous practical domains, including control systems, capacitor modeling, circuit analysis, and control theory [11,12].

The Riemann-Liouville definition can be expressed mathematically as:

$$f(t) = \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \quad (2)$$

It is noteworthy that for real-world applications, particularly those in physical and engineering sciences, the Grünwald-Letnikov and Riemann-Liouville formulations are considered equivalent [13-15].

III. APPROXIMATION METHODS OF A FRACTIONAL ORDER TRANSFER

The literature indicates that numerous approximation methods exist for fractional-order transfer functions. Each available approximation offers specific advantages over others with respect to certain characteristics. Determining which approximation yields optimal results is challenging, as several factors influence the relative performance of any method, including differentiation order, frequency behavior, and time response characteristics. Various approximations are examined here through comparative analysis [16], [17]. These approximations can be categorized into two distinct domains: frequency domain (s-domain) and time domain (z-domain). Approximations in the frequency and time domains are also referred to as continuous and discrete approximations, respectively [18-20].

III.1 OUSTALOUP METHOD

The Oustaloup method is based on the function approximation from as;

$$G_f(s) = S^{\alpha} \quad , \alpha \in R^+ \quad (3)$$

By taking into account the rational function [21-23]:

$$G_f(s) = K \prod_{k=1}^N \frac{s + w'_k}{s + w_k} \quad (4)$$

However, the poles, zeros, and gain can be evaluated as;

$$w'_k = w_b \cdot w_u^{(2k-1-\gamma)/N}, \quad w_k = w_b \cdot w_u^{(2k-1+\gamma)/N}, \quad K = w_h^\gamma$$

Where w_u represents the unity gain in frequency and the central frequency in a geometrically distributed frequency band. Let $w_u = \sqrt{w_h w_b}$, where w_h and w_b represents the upper and lower frequencies, respectively. γ and N are the orders of derivative and filter, respectively.

III.2 MATSUDA METHOD

Matsuda's method employs the continued fraction expansion (CFE) technique [23], which enables the approximation of an irrational function through a rational representation. Given that the selected points are denoted as sk , where $k = 0, 1, 2, \dots, N$, the approximation is expressed in the following form:

$$\hat{G}(s) = a_0 + \frac{s - \omega_0}{a_1 + \frac{s - \omega_1}{a_2 + \frac{s - \omega_2}{a_3 + \dots}}} \quad (5)$$

$$\text{Where } a_i = f(\omega_i), f_0(\omega) = G(s), f_{i+1}(s) = \frac{s - \omega_i}{f_i(s) - a_i}$$

III.3 CHAREF METHOD

The Charef 's transfer method given in [21,22] is based on approximation of the function as;

$$G_f(s) = s^\alpha, \quad \alpha \in R^+ \quad (6)$$

These processes in the frequency domain can be described by an approximation in the Laplace domain such as:

$$G_f(s) = s^\alpha \approx \left(1 + \frac{s}{P_T}\right) \approx \frac{\prod_{i=0}^N (1 + \frac{s}{P_i})}{\prod_{i=0}^{N-1} (1 + \frac{s}{Z_i})} \quad (7)$$

$$\text{Where } P_i = (ab)^i \cdot P_0, \quad i = 1, 2, 3, \dots, N$$

$$Z_i = (ab)^i \cdot a \cdot P_0, \quad i = 2, 3, \dots, N - 1$$

$$\text{and } P_0 = P_T \cdot 10^{\frac{ep}{20\beta}}, \alpha = 10^{\frac{ep}{10(1-\beta)}}, b = 10^{\frac{ep}{10\beta}}, \beta = \frac{\log(a)}{\log(a.b)}$$

IV. FRACTIONALIZATION APPROACH

The proposed approximation strategy design utilizing the "fractionalization" approach is illustrated in the following Figure 2.

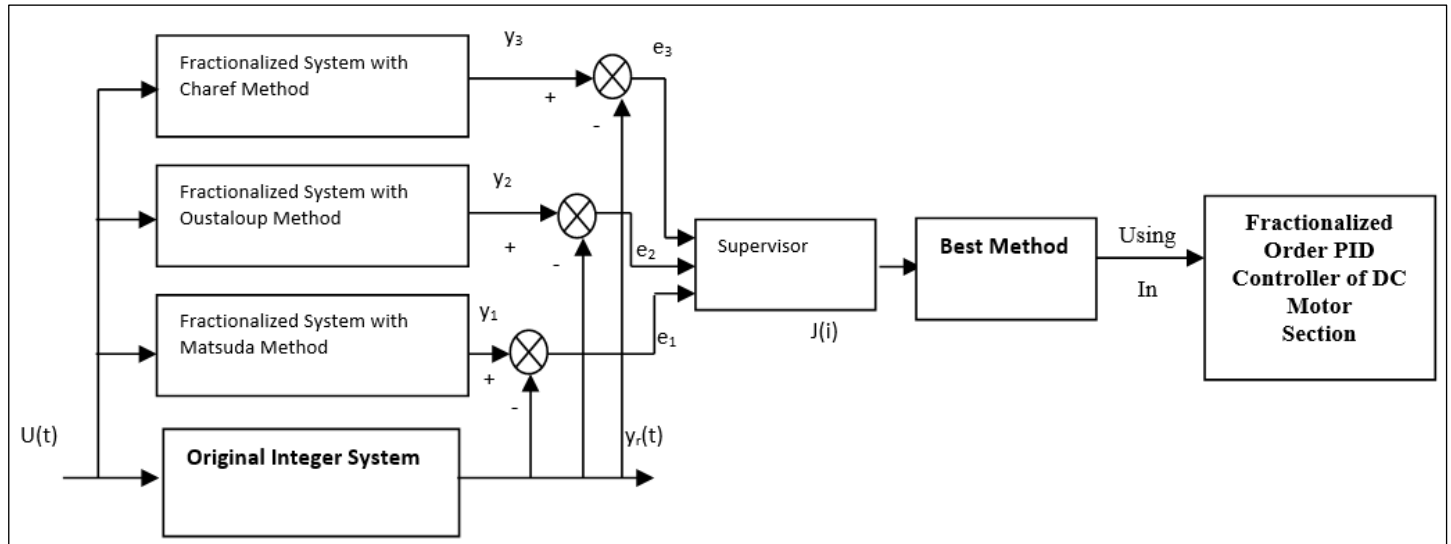


Figure 2: Proposed Approximation strategy design using " Fractionalization" approach.

Source: Authors, (2026).

The supervisor architecture, given in [24], is illustrated by Figure 3.

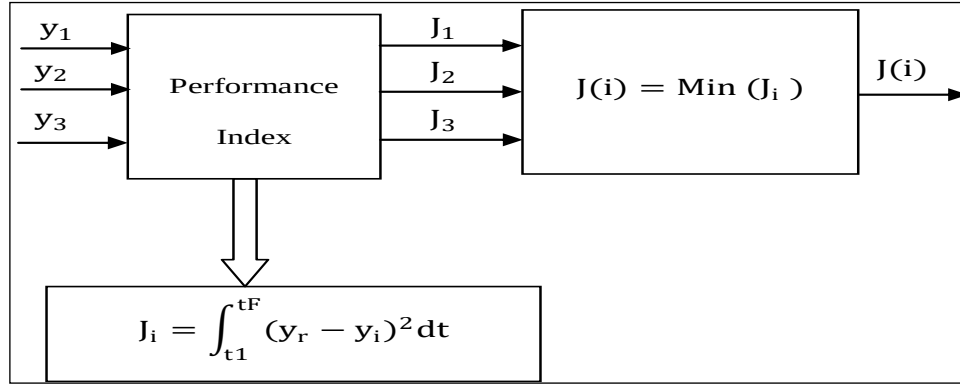


Figure 3: Supervisor Architecture.
Source: Authors, (2026).

Figure 4 shows the fractionalized second Order system.

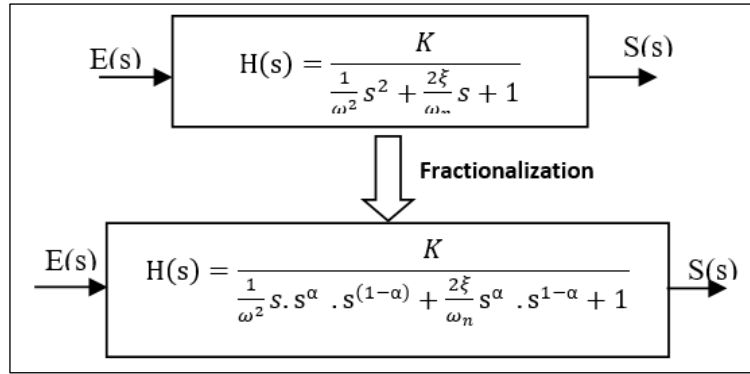


Figure 4: Proposed Fractionalized Second Order system.
Source: Authors, (2026).

The second Order system is converted to a Fractionalized second Order system as follow:

$$H(s) = \frac{K}{\frac{1}{\omega^2} s^2 + \frac{2\xi}{\omega_n} s + 1} = \frac{K}{\frac{1}{\omega^2} s^\alpha \cdot s^{(1-\alpha)} + \frac{2\xi}{\omega_n} s^\alpha \cdot s^{1-\alpha} + 1} \quad (8)$$

The Original Integer Second order System is:

$$H(s) = \frac{2}{s^2 + 3s + 100} \quad (9)$$

Figure 5 show the steps responses comparison between the original second order system and the approximated fractionalized second order system with Oustaloup, Matsuda and Charef approximations methods for $\alpha=0.3$.

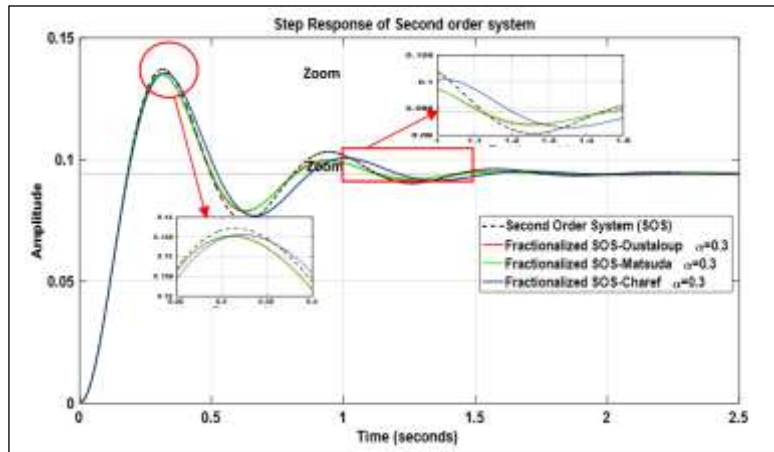


Figure 5: Step response comparison of the original second order system with the fractionalized second order system approximated by Oustaloup, Matsuda, and Charef approximation methods ($\alpha = 0.3$).
Source: Authors, (2026).

The performance Index used by the supervisor defined to evaluate the performance of various approximations methods, it is given by the following equation (10):

$$J_i = \int_{t_1}^{t_F} (y_r - y_i)^2 dt \quad (10)$$

where $i = 1, 2, 3$ for three errors approximating methods (Oustaloup, Matsuda and Charef respectively):

$$e_1 = y_r - y_1, e_2 = y_r - y_2 \text{ and } e_3 = y_r - y_3$$

The criterion for quadratic error when we use the steps responses of second order system are given in Table1:

Table 1: Criterion of quadratic error for steps responses Second order system.

$(\alpha, 1-\alpha)$	(0.1,0.9)	(0.2,0.8)	(0.3,0.7)	(0.4,0.6)	(0.5,0.5)
J_1	0.0087	0.007	0.00549	0.003	0.002
J_2	0.0061	0.005	0.00693	0.002	0.001
J_3	0.0035	0.002	0.0092	0.011	0.1027
Min (J_i)	Charef	Charef	Oustaloup	Matsuda	Matsuda

Source: Authors, (2026).

we remark in the table 1 that when we use the second order system:

The frequency response fitting by Oustaloup, Matsuda and the Charef filter are likewise good, as can be observed. Charef approximation method is the best when $\alpha=0.1$ and $\alpha=0.2$.

For $\alpha=0.3$ the selected method is Oustaloup approximation.

Matsuda method is the best when $\alpha=0.4$ and $\alpha=0.5$.

V. THE FRACTIONALIZED ORDER PID CONTROLLER

The feedback control loop of a fractionalized of the integer order system is shown in Figure 6.

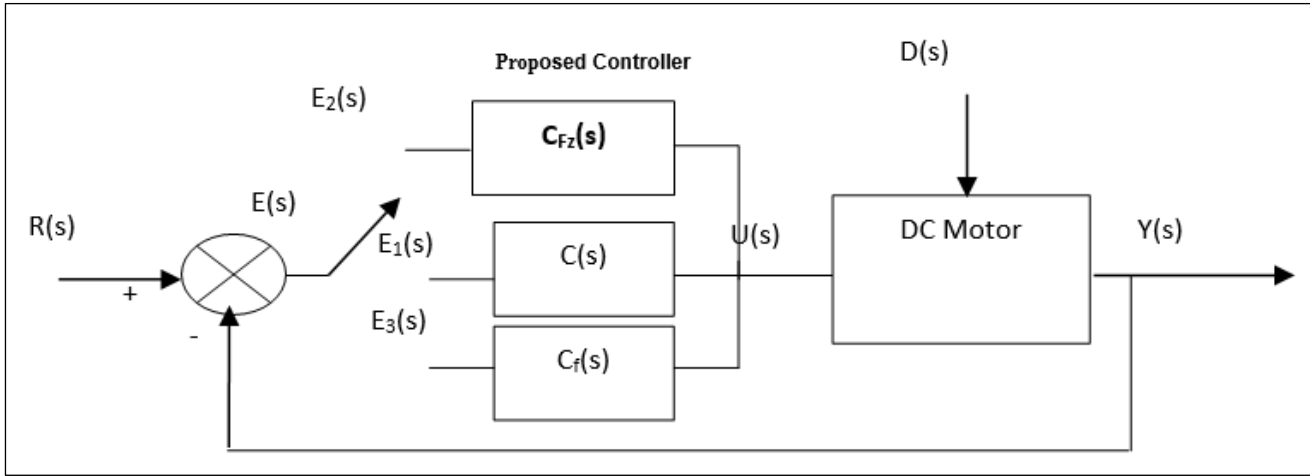


Figure 6: Fractionalized order PID Controller of DC Motor using the best approximation method obtained in Figure 3.

Source: Authors, (2026).

Where, $C_{Fz}(s)$: Fractionalized Controller Transfer Function

The Fractionalized of the integer-order PID controller to be designed is in the following form:

$$C_{Fz}(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right)$$

$$C_{Fz}(s) = \frac{1}{s} \left(\frac{(k_p T_d s^2 + k_p T_i s + k_p)}{T_i} \right)$$

$$C_{Fz}(s) = \frac{1}{s^\alpha} \frac{1}{s^{(1-\alpha)}} \left(\frac{(k_p T_d s^2 + k_p T_i s + k_p)}{T_i} \right) \quad (11)$$

Where, $0 < \alpha < 1$.

V.1 MODELING OF DC MOTOR

The model of the DC Motor is shown in Figure 1. The position $q(t)$, the controlled variable, will be governed by the applied voltage V_a . The angular velocity $w(t)$ is the controlled variable for speed control, and the transfer function takes the following form [23-27]:

$$G_{DC-motor}(s) = \frac{w(s)}{V_a(s)} = \frac{K_m}{(L_a s + R_a)(J s + b) + K_b K_m} \quad (12)$$

But for a lot of DC motors, the armature's time constant $\tau_a = \frac{L_a}{R_a}$ is insignificant, hence the model can be reduced to:

$$G_{DC-motor}(s) = \frac{w(s)}{V_a(s)} = \frac{K_m}{R_a(J s + b) + K_b K_m} = \frac{\frac{K_m}{R_a b + K_b K_m}}{\tau s + 1} = \frac{K_{DC-motor}}{\tau s + 1}$$

where $\tau = \frac{R_a J}{R_a b + K_b K_m}$ and $K_{DC-motor} = \frac{K_m}{R_a b + K_b K_m}$

The General model of a DC Motor is given by the following figure:

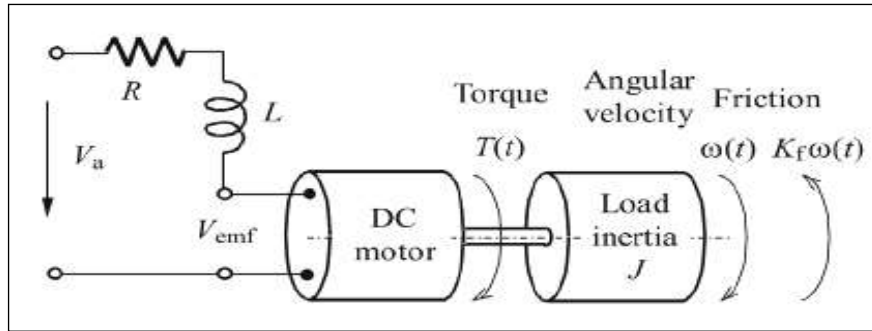


Figure 7: General model of a DC Motor.

Source: Authors, (2026).

The proposed identified model (DC Motor voltage-speed) [24]:

$$G_{DC-motor}(s) = \frac{K_{DC-motor}}{\tau s + 1} = \frac{0.25}{1.45 s + 1} \quad (13)$$

V.2 DISTURBANCE REJECTION IN PID CONTROL

This example shows the comparison of three DC motor control techniques (PID), Fractional PID and Fractionalized PID) for tracking set point commands and reducing sensitivity to load disturbances

The identified model for DC Motor voltage-speed is given by [5]:

$$G_{DC-motor}(s) = \frac{K_{DC-motor}}{\tau s + 1} = \frac{0.25}{1.45 s + 1} \quad (14)$$

Figure 7, Figure 8 and Figure 9 shows the speed of DC Motor using the Integer adaptive PID Controller for $\alpha=0.1$, $\alpha=0.3$ and $\alpha=0.5$ respectively with the following optimized parameters values:

$$k_p = 54.67, k_i = 45.81, k_d = 18.38:$$

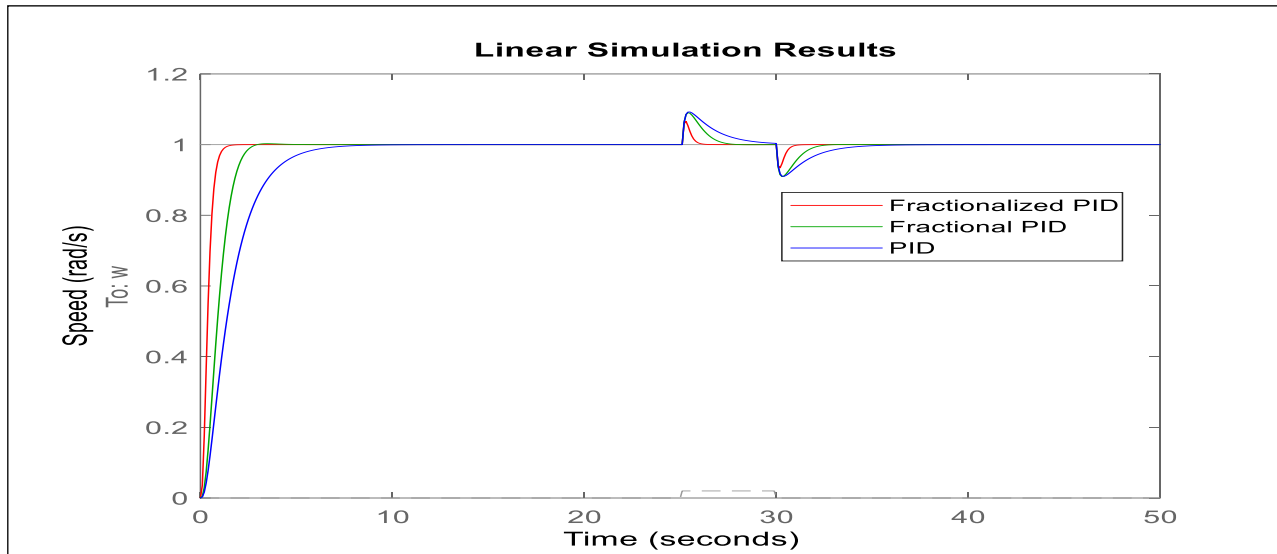


Figure 7: Speed of the DC Motor using the PID, Fractional PID ($\alpha=0.1$) and Fractionalized PID Controllers with disturbance rejection.
Source: Authors, (2026).

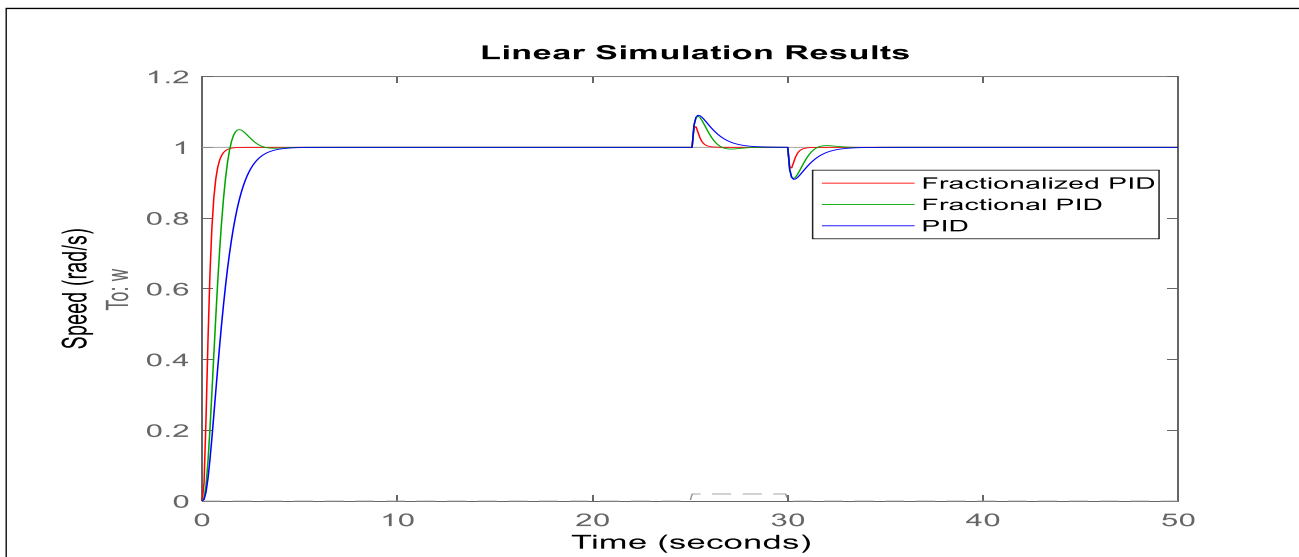


Figure 8: Speed of the DC Motor using the PID, Fractional PID ($\alpha=0.3$) and Fractionalized PID Controllers with disturbance rejection.
Source: Authors, (2026).

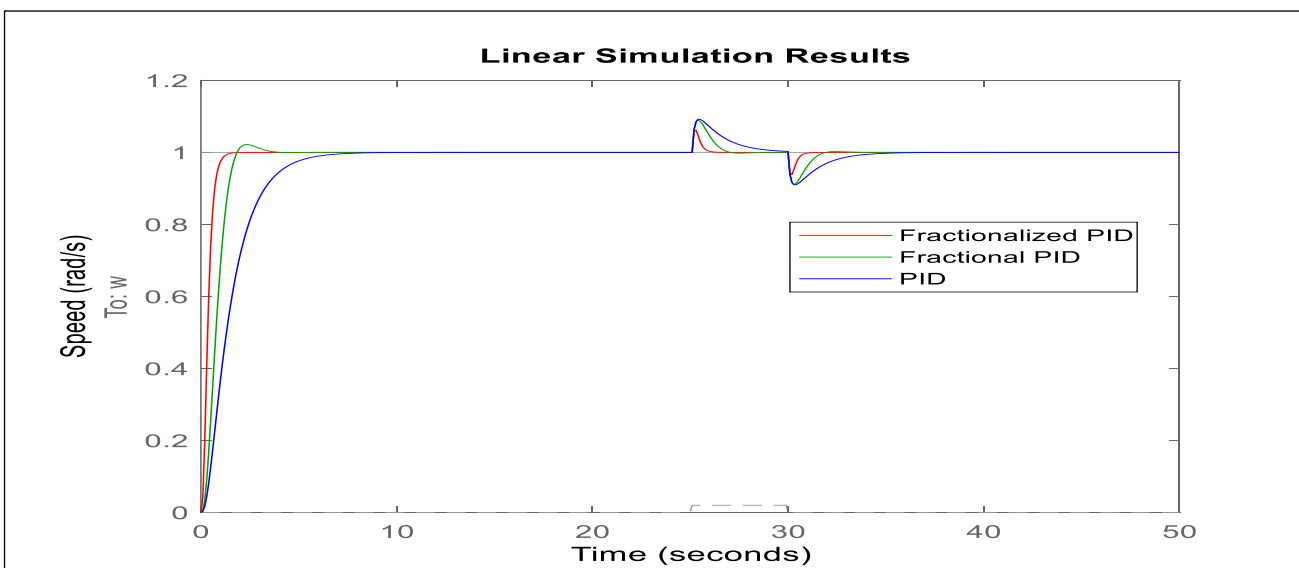


Figure 9: Speed of the DC Motor using the PID, Fractional PID ($\alpha=0.5$) and Fractionalized PID Controllers with disturbance rejection.
Source: Authors, (2026).

The performance analysis of the proposed fractional order adaptive PID controller and the classical adaptive PID controller is given by the following table:

Table 2: Comparative analysis of transient response performance of DC Motor

Controllers	Overshoot [%]	Setting time [s]	Rise time [s]	Criterion of quadratic error J
PID	0.0012	4.1	1.75	0.22
Fractional PID	0.0026	2.1	1.1	0.12
Fractionalized PID [$\alpha = 0.1$]	0.0000	1.14	0.422	0.026
Fractionalized PID [$\alpha = 0.3$]	0.0000	1.16	0.423	0.025
Fractionalized PID [$\alpha = 0.5$]	0.0000	1.15	0.421	0.027

Source: Authors, (2026).

We remark that the fractionalized PID Controller for DC MOTOR system give the good improvement of overshoot, setting time , rise time and mean absolute error comparatively to the Integer and the fractional PID Controller results. That means that proposed controller is the most stable system in terms of frequency response criterion.

VI. CONCLUSIONS

This paper introduces an innovative methodology for evaluating and comparing various approximation methods of fractional-order systems, as well as assessing disturbance rejection capabilities. The approach involves incorporating fractional-order filters into the original system design while maintaining the integrity of the overall equivalent transfer function. Building upon the fundamental concept of integrating fractional-order operators into the original integer-order system, the fractionalization approach demonstrates its effectiveness in identifying the optimal approximation method. Furthermore, it serves as a robust comparative tool for evaluating different approximation techniques for fractional systems, specifically the Oustaloup, Matsuda, and Charef approximations.

The methodology also exhibits superior disturbance rejection performance in fractionalized PID control applications for DC motors. These findings have been validated through a comprehensive set of simulation examples that demonstrate the practical effectiveness of the proposed approach. Future research will focus on extending this fractionalization methodology to broader classes of integer-order functions. The objective is to enhance noise rejection capabilities and improve robustness across various control system architectures and applications. This generalization is expected to provide greater flexibility and wider applicability of the fractionalization approach in diverse engineering contexts.

VII. AUTHOR'S CONTRIBUTION

Conceptualization: Benzenati Rahima, Bensafia Yassine and Lehouche Hocine.

Methodology: Benzenati Rahima, Bensafia Yassine and Lehouche Hocine.

Investigation: Benzenati Rahima, Bensafia Yassine and Lehouche Hocine.

Discussion of results: Benzenati Rahima, Bensafia Yassine and Lehouche Hocine.

Writing – Original Draft: Bensafia Yassine.

Writing – Review and Editing: Benzenati Rahima, Bensafia Yassine and Lehouche Hocine.

Resources: Benzenati Rahima, Bensafia Yassine and Lehouche Hocine.

Supervision: Benzenati Rahima, Bensafia Yassine and Lehouche Hocine.

Approval of the final text: Benzenati Rahima, Bensafia Yassine and Lehouche Hocine.

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