

RESEARCH ARTICLE

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FERMATEAN FUZZY GRAPH IN DECISION MAKING MODELS THEORY

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ARTICLE INFO

Article History

Received: February 13, 2026

Reviewed: April 21, 2026

Accepted: May 21, 2026

Published: June 30, 2026

Keywords:FFS,
FFG,
MCDM,
Uncertainty modelling,
Hesitation degree, Decision-
support systems,
Similarity measure.

ABSTRACT

Fermatean Fuzzy Graph (FFG) is a prominent mathematical model effective in conveying uncertainty, hesitation and dual membership characteristics in decision-making domains where traditional models often exhibit ineptness in handling complex ambiguities. Thereby, an extensive investigation of Fermatean fuzzy graphs within decision-making frameworks is executed in this study to emphasise their theoretical underpinnings, structural attributes and practical implications. Through the extension of Fermatean fuzzy set theory, the proposed model represents each vertex and edge with cubic membership, non-membership and hesitation degrees. This in turn results in the enrichment of the descriptive capacity of relational data structures. Key operations including union, intersection, complement and Cartesian product are formulated in the study under Fermatean conditions, and their algebraic and topological features are then thoroughly discussed. In order to support multi-criteria evaluations in contexts characterised by ambiguity and contradicting information, the developed decision-making model incorporates FFG structures with weighted aggregation operators and similarity measures. The superior ability of FFGs to handle uncertainty with enhanced precision and decision dependability is demonstrated by comparison with intuitionistic and Pythagorean fuzzy graph models.



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I. INTRODUCTION

Professionals from a wide range of fields deal with complicated, multi-criteria scenarios in modern decision environments that are marked by ambiguity, insufficient information and conflicting objectives. Under these conditions, Decision-Making (DM) is a cognitive process of reasoning and judgement rather than just a computational task [1], [2]. It seeks to choose the best option that strikes a fair balance between a number of conflicting criteria [3]. However, on the downside, the lines between ideal and non-ideal decisions are frequently blurred by real-world uncertainty, rendering traditional deterministic methods inept [4]. Multi-Criteria Decision-Making (MCDM) techniques have been created as organised analytical structures to address this ambiguity [5-7]. These methods allow for the logical comparison and ranking of alternatives according to both quantitative and qualitative criteria. Among the various MCDM techniques, the Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS) [8], Analytic Hierarchy Process (AHP) [9], VIKOR [10] and ELECTRE [11] have been recognized as effective approaches for ranking and selecting alternatives. By measuring each alternative's distance from ideal and anti-ideal points, these models identify the optimal solution. However, their reliance on crisp data limits their effectiveness in uncertain or hesitant settings where decision-maker's viewpoints are vague and often contradictory. Consequently, the combination of fuzzy logic with MCDM techniques proved a significant breakthrough as it is effective in combining algorithmic rigour with human reasoning [12]. The foundation of uncertainty modelling was established by Zadeh's Fuzzy Set Theory (FST), which introduced the idea of partial membership between 0 and 1 to measure vagueness. However, decision-makers frequently feel both agreement and disagreement with a certain attribute at the same time while making decisions in real-world assessments [13]. Atanassov developed Intuitionistic Fuzzy Sets (IFS) to account for this dual nature. IFS includes an indeterminacy measure in addition to membership and non-membership degrees [14]. Subsequently, Yager presented Pythagorean Fuzzy Sets (PFS), which provide more flexibility than IFS by limiting the squared sum of membership and non-membership degrees to unity. Despite broadening the uncertainty domain, PFS models were inadequate in situations where membership and non-membership values were large, resulting in overly constrained hesitation ranges [15].

For subduing this, Senapati and Yager created FFS, which feature a cubic constraint that allows for a more flexible and expansive representation of uncertainty. Because the FFS model preserves mathematical validity while permitting decision-makers to communicate stronger but conflicting ideas, it effectively captures severe uncertainty [16]. As a result, FFS has drawn interest in fields where uncertainty and hesitancy predominate, like risk management, supplier selection, medical diagnosis and environmental evaluation. The FFG model is a recent development that builds on this progress by extending the Fermatean concept into relational domains. An FFG uses Fermatean membership and non-membership degrees to define each vertex, which represents an alternative and each edge, which indicates the relationship or similarity between alternatives. In contrast to traditional FFS or PFS designs, this structure enables the model to capture not just uncertainty in discrete criteria but also the interdependencies across alternatives. By facilitating multi-level reasoning and uncertainty transmission among interconnected entities, the FFG offers decision systems a relationally enhanced perspective. Using FFG to develop an integrated decision-support model that analyses relational, hesitant and ambiguous data with improved interpretability serves as the motivation for this research. By integrating similarity computations with Fermatean aggregation in a graph-theoretic structure, the proposed research seeks to provide well-rounded choice results that account for both individual and collective uncertainty.

II. LITERATURE REVIEW

The conceptual evolution of fuzzy decision systems has followed a logical progression from Fuzzy Sets (FS) to IFS, PFS and most recently, FFS. Although uncertainty representation was made possible by [17] groundbreaking work in 1965, the original FS framework could only evaluate membership on a single dimension. In (1999) [18] developed IFS to address this, including a hesitation index to account for partial preferences and a complementing non-membership component. In MCDM contexts, IFS techniques promptly came into use in fields like risk assessment, medical diagnostics and supplier evaluation. As decision complexity grew, by [19] extended IFS into PFS, employing the quadratic relation $\mu^2 + \nu^2 \leq 1$ to enlarge the uncertainty zone and offer improved flexibility. A number of research efforts incorporated PFS-based models into established MCDM methodologies. For instance, in (2018) [20] employed Hausdorff metric-based PFS distances, in [21] used cosine similarity measures for fuzzy TOPSIS, and [22] proposed generalized distance metrics for ranking alternatives. PFS techniques have been limited in expressing extreme hesitation, where both degrees of belief and disbelief are simultaneously high, despite their broad application. In [23] introduced the cubic constraint $\mu^3 + \nu^3 \leq 1$ in their FFS structure to overcome this constraint. By capturing greater levels of hesitancy and uncertainty, the Fermatean model established itself as a sophisticated extension of IFS and PFS. Subsequent research expanded FFS to include similarity metrics, correlation coefficients and aggregation operators for a variety of decision-making applications.

In [24] utilized FFS in medical decision analysis, while in [25] applied Fermatean fuzzy weighted sum methods in product evaluation. These investigations show that the cubic formulation provided better decision accuracy and modelling flexibility. The graph-based extension of fuzzy systems further advances the field. Fuzzy Graphs (FGs), Intuitionistic Fuzzy Graphs (IFGs) and Pythagorean Fuzzy Graphs (PFGs) were developed to incorporate relational structures among decision elements. The limitations of their underlying fuzzy theories were still present in these models. This was addressed with the introduction of the FFG, which combines the structural resilience of graph theory with the expressive capacity of FFS. Relational analysis under higher-order uncertainty is made possible by FFGs, which use Fermatean numbers to define vertex and edge relationships. In multi-criteria fields including social influence analysis, renewable energy planning and logistics optimisation, this model demonstrated its worth. Recent research has investigated quantifying relational proximity between vertices using Fermatean distance and similarity functions, which increases the accuracy of MCDM systems. To ensure that both determinacy and hesitation are taken into account in the final decision, FFG-based similarity operators, for instance, permit alternative ranking based on aggregated uncertainty. Additionally, the FFG supports both qualitative and quantitative evaluations by offering a unified platform for integrating multi-attribute information. FFG is especially applicable to modern decision issues with contradictory, interdependent and conflicting data because of these advantages.

II.1 RESEARCH GAP AND OBJECTIVES

The integration of Fermatean theory with graph-based decision modelling has not been extensively studied, despite the fact that many studies have improved MCDM frameworks employing IFS, PFS and FFS models. While conventional fuzzy systems mainly handle uncertainty at the level of individual criteria, they are unable to capture inter-criterion and inter-alternative relationships. Similarly, the majority of distance-based decision-making techniques, such as fuzzy TOPSIS and PFS-VIKOR, are restricted to vector-based assessments and disregard relational dependencies. Their applicability in complex systems, such energy dispatch, transportation logistics, healthcare systems, and intelligent urban planning, where decisions are inevitably networked or interactive, is limited by this gap. The proposed research suggests a FFG-driven decision framework that combines relational modelling with Fermatean uncertainty principles in order to address this research gap. The following are the objectives of this research,

- To use FFG to build a mathematical model of decision alternatives that accounts for relational and nodal uncertainty.
- To create similarity and Fermatean aggregation operators to assess the relative proximity of selection options.
- To combine all metrics into a single decision layer that calculates normalised similarity indices and determines the optimal alternative with maximum confidence.
- To demonstrate the superiority of the proposed FFG model compared with existing IFS- and PFS-based MCDM frameworks in handling ambiguous, interdependent information

The goal of this technique is to develop a thorough uncertainty-aware decision model that can generate precise, consistent, and comprehensible results in settings where uncertainty, conflict and hesitation coexist.

III. THEORETICAL BACKGROUND

III.1 FERMEATEAN FUZZY SET (FFS)

In an effort to effectively manage hesitancy and contradiction in uncertain information, fuzzy logic has undergone multiple extensions, starting with Zadeh's fuzzy set and progressing with Atanassov's Intuitionistic Fuzzy Set (IFS) and Yager's Pythagorean Fuzzy Set (PFS). Nevertheless, the ability of both IFS and PFS to depict scenarios involving high levels of simultaneous belief and disbelief remains confined. The concept of FFS introduced by Senapati and Yager improves these prior models with the substitution of a cubic condition for the quadratic constraint of PFS. Through this improvement, the feasible region of membership and non-membership values are broadened substantially, thereby allowing for richer and more balanced uncertainty modelling for relational, control and decision-making systems.

III.1.1 Definition

Let X denote a non-empty universe of discourse. A Fermatean fuzzy set I on X is expressed as,

$$I = \{(x, \mu_I(x), v_I(x)) : x \in X\} \quad (1)$$

Where, $\mu_I(x) : X \rightarrow [0,1]$ and $v_I(x) : X \rightarrow [0,1]$ represent the degrees of membership and non-membership, respectively. These values satisfy the cubic constraint,

$$0 \leq (\mu_I(x))^3 + (v_I(x))^3 \leq 1 \quad (2)$$

The hesitation degree of x is derived as,

$$\pi_I(x) = \sqrt[3]{1 - (\mu_I(x))^3 - (v_I(x))^3} \quad (3)$$

which measures the residual uncertainty associated with the element. A pair (μ, v) that fulfils (2) is known as Fermatean Fuzzy Number (FFN). The extreme FFNs are $x^+ = (1,0)$ indicating full membership and $x^- = (0,1)$ representing full non-membership.

III.1.2 Operational Laws

For any two FFNs $\rho_1 = (\mu_1, v_1)$ and $\rho_2 = (\mu_2, v_2)$ and a positive scalar $\lambda > 0$, the algebraic operations are defined as follows,

$$\rho_1 \oplus \rho_2 = \left(\sqrt[3]{\mu_1^3 + \mu_2^3 - \mu_1^3 \mu_2^3}, v_1 v_2 \right) \quad (4)$$

$$\rho_1 \ominus \rho_2 = \left(\mu_1 \mu_2 \sqrt[3]{v_1^3 + v_2^3 - v_1^3 v_2^3} \right) \quad (5)$$

$$\lambda_\rho = \left(\sqrt[3]{1 - (1 - \mu^3)^\lambda}, v^\lambda \right) \quad (6)$$

$$\rho^\lambda = \left(\mu^\lambda, \sqrt[3]{1 - (1 - v^3)^\lambda} \right) \quad (7)$$

$$\rho^c = (v, \mu) \quad (8)$$

Consistent computation in complex modelling systems is made possible by these procedures, which maintain the cubic constraint and guarantee algebraic closure within the FFS domain.

III.1.3 Distance and Similarity Measures

For two FFNs $\rho_1 = (\mu_1, v_1)$ and $\rho_2 = (\mu_2, v_2)$, the Fermatean Fuzzy Distance (FFD) is defined as,

$$FFD(\rho_1, \rho_2) = \sqrt{\frac{1}{2}[(\mu_1^3 - \mu_2^3)^2 + (v_1^3 - v_2^3)^2 + (\pi_1^3 - \pi_2^3)^2]} \quad (9)$$

Where, $\pi_i = \sqrt[3]{1 - \mu_i^3 - v_i^3}$ ($i = 1,2$). The Fermatean Fuzzy Similarity (FFS) measure, expressing the closeness between two FFNs, is obtained as,

$$FFS(\rho_1, \rho_2) = 1 - FFD(\rho_1, \rho_2) \quad (10)$$

While lower values signify greater divergence, a similarity score close to 1 suggests a high level of agreement between the two fuzzy evaluations.

III.1.4 Interpretation and Relevance

Compared to intuitionistic and Pythagorean frameworks, the cubic formulation of FFS offers greater flexibility by expanding the acceptable uncertainty zone. It captures instances in which an element exhibit both strong membership and non-membership inclinations at the same time, which is typical in scenarios involving uncertainty or contradicting information when making decisions.

The proposed FFG model in this work is mathematically supported by the FFS. FFNs, whose membership, non-membership and hesitation degrees jointly reflect the confidence, opposition, and vagueness inherent in relational data, are used to parameterise each vertex and edge in the graph. These features allow the model to evaluate similarity and conduct fine-grained reasoning in hesitant or uncertain contexts.

III.2 FERMEATEN FUZZY GRAPH (FFG)

A useful mathematical tool for illustrating pairwise interactions between items is graph theory. Each vertex and edge in a classical fuzzy graph has a unique membership degree that indicates the existence or strength of a relation. Nevertheless, uncertainty or contradicting information cannot be handled by such models. Figure 1 provides the basic fuzzy graph representation. The FFG, a model that describes membership, non-membership and indeterminacy simultaneously through cubic interactions, is the outcome of integrating FFS theory into graph topologies. This addition strengthens the graph's descriptive capabilities, which makes it especially appropriate for complex decision-making situations involving interconnected entities that exhibit hesitation, contradiction or uncertainty.

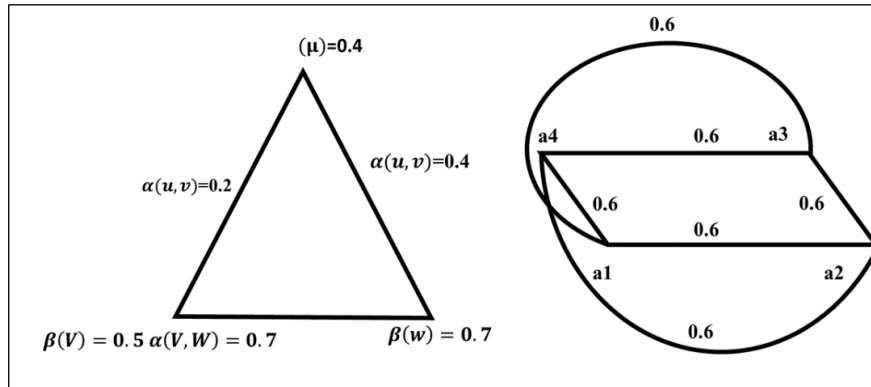


Figure 1: Basic Fuzzy Graph representations showing single-membership relations between vertices.
Source: Authors, (2026).

III.2.1 Definition of Fermatean Fuzzy Graph

Let X be a universal set and $P = (\alpha_P, \beta_P)$ represent an FFS on X . A FFG on X is denoted by $G = (P, Q)$, where $Q = (\alpha_Q, \beta_Q)$ is a Fermatean fuzzy relation on $X \times X$. The structure satisfies the following conditions,

$$\alpha_Q(u, v) \leq \min\{\alpha_P(u), \alpha_P(v)\} \quad (11)$$

$$\beta_Q(u, v) \geq \max\{\beta_P(u), \beta_P(v)\} \quad (12)$$

$$(\alpha_Q(u, v))^3 + (\beta_Q(u, v))^3 \leq 1 \quad (13)$$

For all $u, v \in X$. Here, $\alpha_Q(u, v)$ and $\beta_Q(u, v)$ denote the membership and non-membership degrees of the edge connecting vertices u and v . The hesitation associated with the edge is obtained as,

$$\pi_Q(u, v) = \sqrt[3]{1 - (\alpha_Q(u, v))^3 - (\beta_Q(u, v))^3} \quad (14)$$

A richer uncertainty domain is offered by this cubic formulation as opposed to intuitionistic or Pythagorean fuzzy graphs. Consider a simple Fermatean fuzzy graph $G = (P, Q)$ where $P = \{a_1, a_2, a_3, a_4\}$ and the vertex set P is characterized by,

$$\alpha_P(a_1) = 0.3, \beta_P(a_1) = 0.5; \alpha_P(a_2) = 0.4, \beta_P(a_2) = 0.6$$

The corresponding edge set $Q = \{(a_1, a_2), (a_2, a_3)\}$ probably have,

$$\alpha_Q(a_1, a_2) = 0.2, \beta_Q(a_1, a_2) = 0.4; \alpha_Q(a_2, a_3) = 0.3, \beta_Q(a_2, a_3) = 0.5$$

Each pair (α_Q, β_Q) satisfies the cubic condition, and their indeterminacy values can be obtained using (14). This example illustrates the way different levels of uncertainty and reliability can be modelled by both vertices and edges in an FFG. The structure of the constructed FFG is depicted in Figure 2, where both vertex and edge parameters satisfy the cubic constraint. The interconnection among multiple vertices in the proposed FFG is depicted in Figure 3.

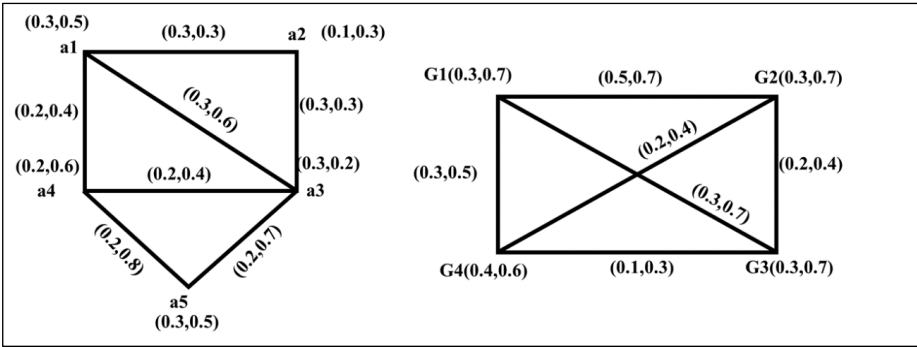


Figure 2: FFG illustrating vertex and edge membership-non-membership pairs that satisfies the cubic constraint.
Source: Authors, (2026).

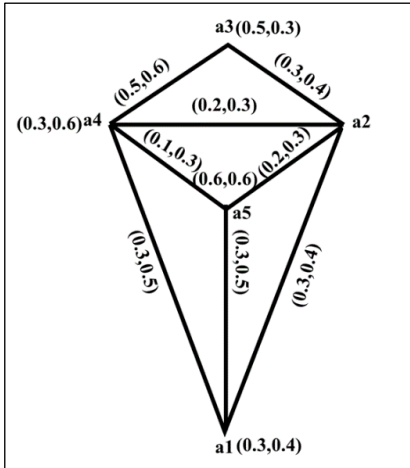


Figure 3: FFG illustrating multi-vertex relationships under cubic membership-non-membership constraints.
Source: Authors, (2026).

III.2.2 Structural Operations on FFGs

FFGs can go through a number of structural operations, just like conventional graphs, which makes it possible to analyse big, interconnected systems,

- **Union of Two FFGs:**

For graphs $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$, the union $G_1 \cup G_2$ is defined as $\alpha_{P_1 \cup P_2}(x) = \max\{\alpha_{P_1}(x), \alpha_{P_2}(x)\}$, $\beta_{P_1 \cup P_2}(x) = \max\{\beta_{P_1}(x), \beta_{P_2}(x)\}$. Similar conditions apply to edge functions.

- **Cartesian Product:**

The Cartesian product of two FFGs, $G_1 \times G_2$, is constructed by combining vertex and edge sets while preserving Fermatean relationships between them. The cartesian product of FFGs is presented in Figure 4.

- **Composition of FFGs:** Given $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$, the composition $G_1 \circ G_2$ links vertices across graphs via the intermediate relations defined by Q_1 and Q_2 , maintaining cubic membership–non-membership constraints.

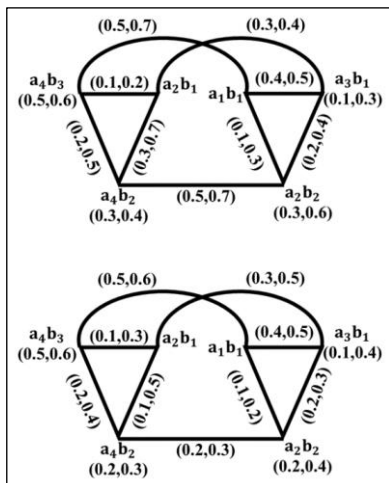


Figure 4: Cartesian product of FFGS.
Source: Authors, (2026).

These procedures make it possible to represent multi-layer decision networks hierarchically, which is crucial for applications such as energy network optimisation, medical diagnosis, and supply chain selection.

III.2.3 Relevance in Decision-Making

Uncertain interdependencies between several elements, such as criteria, options, or system nodes, can be represented using the FFG structure. Each edge uses Fermatean fuzzy degrees to quantify the relational influence between two components. These relationships are converted into adjacency matrices when incorporated into an MCDM framework, which enables the ranking, scoring, and aggregation of alternatives according to their total connectivity and confidence metrics. Mathematically, if S_i and S_j represent decision alternatives, the Fermatean similarity between them is evaluated using,

$$FFS(S_i, S_j) = 1 - FFD(S_i, S_j) \quad (15)$$

Where, $FFD(\cdot)$ is the Fermatean distance. By combining determinacy and hesitation in pairwise comparisons, this similarity metric improves the resilience of decisions.

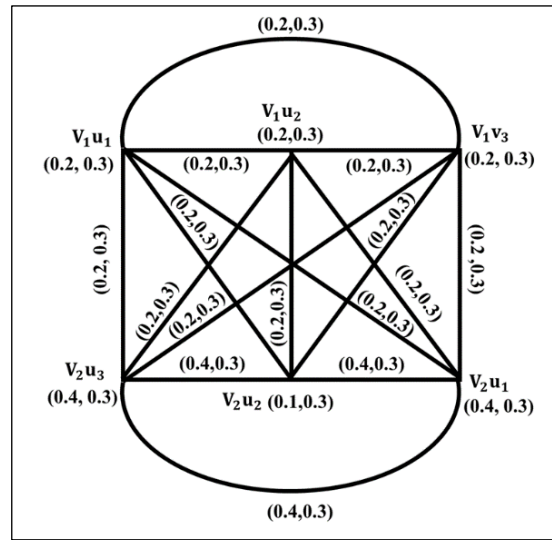


Figure 5: Composition of two FFGs showing relational links formed via intermediate vertices.

Source: Authors, (2026).

The compositional interaction between two FFGs is illustrated in Figure 5, where relational connections are established through intermediate vertex pairs that preserve the cubic membership–non-membership constraints. Compared to intuitionistic or Pythagorean approaches, the FFG model provides a deeper depiction of ambiguous and interdependent decision elements because it forms the relational basis of the proposed framework. It efficiently measures hesitation, handles conflicting information and preserves mathematical consistency in uncertainty-aware computation by representing every node and edge as a Fermatean fuzzy entity.

IV. PROPOSED METHODOLOGY

IV.1 SYSTEM OVERVIEW

The main mechanism for capturing and processing uncertainty in complex relational systems is the FFG model, which forms the basis of the proposed decision-making system. By converting ambiguous and interdependent decision aspects into a network of Fermatean fuzzy vertices and edges, the technology allows for systematic evaluation using reasoning based on similarity and aggregation. The model comprises of four sequential phases, which are,

- Fermatean Graph Construction, where decision entities are represented as vertices and their pairwise interactions as edges, each defined by FFNs (μ, ν, π) that satisfy the cubic constraint $\mu^3 + \nu^3 \leq 1$.
- Aggregation and Similarity computation, which converts each vertex and edge parameter into a weighted adjacency matrix. For evaluating the degree of the relationship between alternatives, these matrices, which encode both determinacy and hesitation are further processed using Fermatean similarity operators.
- In decision layer integration, the most desirable option is determined by combining the aggregated Fermatean data with multi-criteria optimisation.
- Comparison with traditional Intuitionistic and Pythagorean fuzzy graph models is part of the evaluation and validation process, which confirms the resilience and accuracy of the decisions.

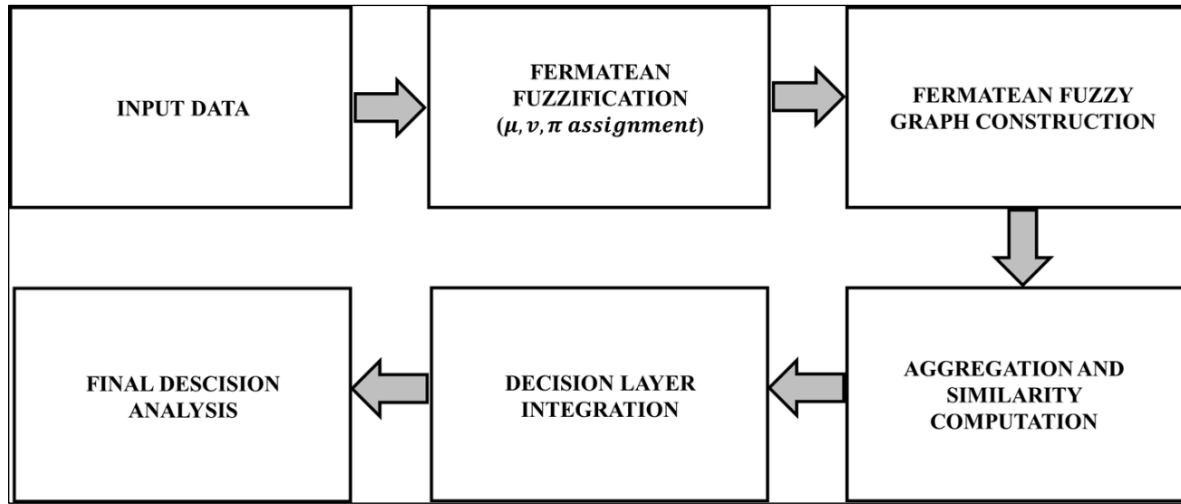


Figure 6: Conceptual workflow of the proposed system.

Source: Authors, (2026).

The entire conceptual workflow of the proposed system is depicted in Figure. The multi-dimensional uncertainty relationships are captured by the constructed FFG after raw decision data undergoes Fermatean fuzzification. This relational data is converted into quantitative decision indices that direct the optimisation process in the ensuing aggregation and similarity stages. Because of its modular design, the system is effective in managing contradictory, hesitant or incomplete information while maintaining computational consistency.

IV.2 FERMATEAN GRAPH CONSTRUCTION

FFG construction constitutes as the most vital stage in the proposed decision-making methodology. This step transforms the decision environment into a relational graph structure where each vertex and edge denotes a unique entity and interaction in an uncertain situation. Each component conveys the degrees of membership, non-membership and hesitation related to its evaluation owing to the cubic Fermatean formulation. Let,

$$G = (P, Q) \quad (16)$$

Denote a FFG, where P specifies vertex set and Q specifies edge set. For a finite collection of vertices $V = \{v_1, v_2, \dots, v_n\}$, each vertex $v_i \in V$ is defined by a Fermatean fuzzy pair,

$$P(v_i) = (\alpha_P(v_i), \beta_P(v_i)), \quad 0 \leq \alpha_P^3(v_i) + \beta_P^3(v_i) \leq 1 \quad (17)$$

Similarly, the edge joining two vertices v_i and v_j is characterized as,

$$Q(v_i, v_j) = (\alpha_Q(v_i, v_j), \beta_Q(v_i, v_j)) \quad (18)$$

Subject to,

$$\alpha_Q(v_i, v_j) \leq \min\{\alpha_P(v_i), \alpha_P(v_j)\} \quad (19)$$

$$\beta_Q(v_i, v_j) \leq \min\{\beta_P(v_i), \beta_P(v_j)\} \quad (20)$$

The associated hesitation degree for each edge is obtained as,

$$\pi_Q(v_i, v_j) = \sqrt[3]{1 - (\alpha_Q(v_i, v_j))^3 - (\beta_Q(v_i, v_j))^3} \quad (21)$$

Ensuring that all Fermatean relations satisfy the cubic constraint.

IV.2.1 Adjacency and Weight Matrices

For computational ease, the FFG's relational structure can be expressed as a matrix. Let A_α and A_β denote the membership and non-membership adjacency matrices, respectively, defined as,

$$A_\alpha = [\alpha_Q(v_i, v_j)]_{n \times n}, A_\beta = [\beta_Q(v_i, v_j)]_{n \times n} \quad (22)$$

From these, the hesitation matrix A_π is derived using (21). The strength, opposition or indeterminacy between any two choice parameters is encoded by each matrix element. A weighted relational matrix W is then formulated as,

$$W = \omega_1 A_\alpha - \omega_2 A_\beta + \omega_3 A_\pi \quad (23)$$

Where, $\omega_1 + \omega_2 + \omega_3 = 1$ and $0 \leq \omega_i \leq 1$ represent the relative importance of each Fermatean component. This weighting system guarantees that determinacy and hesitation are fairly taken into account in the ensuing similarity and aggregation calculations.

IV.2.2 Graph Construction Procedure

The entire graph construction process proceeds through the following steps,

1. Identification of Decision Entities:

Determine the set of alternatives or criteria to be evaluated and assign each as a vertex v_i .

2. Computation of Fermatean Parameters:

For each vertex and edge, estimate the membership α and non-membership β degrees from expert judgement, performance indicators, or normalized quantitative data.

3. Validation of Cubic Constraint:

Verify that all pairs satisfy $\alpha^3 + \beta^3 \leq 1$; adjust values proportionally if violated.

4. Formation of Adjacency Matrices:

Construct A_α, A_β and A_π using Equations (22)-(23).

5. Weight Allocation:

Assign weighting factors based on the desired decision sensitivity toward certainty, contradiction, or hesitation.

6. Graph Visualization:

Generate the network topology $G = (P, Q)$ where nodes correspond to decision variables and edges reflect their Fermatean relations.

For the purpose of ensuring that both determinacy and hesitation are mathematically incorporated inside the decision space, this construction phase converts ambiguous raw data into a structured Fermatean relational network. The model provides high descriptive granularity while maintaining semantic consistency by maintaining the cubic relationship between membership, non-membership and hesitation.

IV.3 AGGREGATION AND SIMILARITY COMPUTATION

The relational information recorded in the FFG is integrated into a quantitative form appropriate for decision evaluation during the aggregation and similarity computation phase. In this step, the vertices and edges are combined using weighted aggregation after their membership, non-membership and hesitation values have been determined. Fermatean similarity indices are then used to determine the degree of resemblance between decision alternatives.

IV.3.1 Fermatean Weighted Aggregation

Multiple qualities or relations frequently convey unequal significance in decision systems. Fermatean aggregation operators are used to get a single value from several fuzzy evaluations in order to account for this variation. Let $X = \{\rho_1, \rho_2, \dots, \rho_n\}$ be a set of Fermatean fuzzy numbers, each defined as $\rho_i = (\mu_i, v_i)$ and satisfying $\mu_i^3 + v_i^3 \leq 1$. The FWA operator is defined as,

$$FWA(\rho_1, \rho_2, \dots, \rho_n) = \left(\sqrt[3]{1 - \prod_{i=1}^n (1 - \mu_i^3)^{w_i}}, \prod_{i=1}^n v_i^{w_i} \right) \quad (24)$$

Where, $w_i \geq 0$ and $\sum_{i=1}^n w_i = 1$. An aggregated Fermatean pair that maintains the cubic constraint is produced by the FWA operator. The aggregated contradiction (disagreement) among the characteristics under consideration is represented by the second term, whereas the first term encapsulates the collective determinacy (agreement). The operator reduces to the Fermatean Arithmetic Mean (FAM) when all criteria are equally weighted,

$$FAM(\rho_1, \rho_2, \dots, \rho_n) = \left(\sqrt[3]{\frac{1}{n} \sum_{i=1}^n \mu_i^3}, \sqrt[3]{\frac{1}{n} \sum_{i=1}^n v_i^3} \right) \quad (25)$$

For creating composite relational indices that reflect overall membership, non-membership and hesitation for every decision alternative, these aggregation operators are applied to each row and column of the adjacency matrices. The Fermatean Fuzzy Similarity metric is used to evaluate the relationship proximity between two alternatives. For two FFNs $\rho_1 = (\mu_1, v_1)$ and $\rho_2 = (\mu_2, v_2)$, the FFD is computed using Equation (9). The corresponding similarity index is then defined using Equation (10). While a lower value implies divergence in decision characteristics, a higher FFS value implies greater resemblance or consistency between the alternatives.

IV.3.2 Computation Process

Using the weighted or arithmetic aggregation operators, the edge-based Fermatean parameters associated with each vertex are combined in input aggregation. This is the first step in the computation process for aggregation and similarity evaluation. As a result, each decision alternative's overall Fermatean characteristics are represented by an aggregated vertex measure $P'(A_i)$. Subsequently, the pairwise similarity evaluation stage computes the degree of resemblance between every pair of alternatives (A_i, A_j) by applying the Fermatean distance and similarity formulations. Every pairwise similarity indices are included in the similarity matrix $S = [FFS(A_i, A_j)]_{n \times n}$ that is created from the resulting values. Each alternative's overall similarity score S_i is determined by taking the mean of its pairwise similarities with every other alternative from this matrix,

$$S_i = \frac{1}{n-1} \sum_{j \neq i} FFS(A_i, A_j) \quad (26)$$

Alternatives are then ranked in descending order of S_i , where a higher value reflects stronger relational closeness and thus greater suitability within the decision framework. In order to provide a consistent basis for comparison and to serve as confidence indicators for the final decision selection, a normalisation phase makes sure that all similarity scores fall within the interval $[0,1]$.

The relational structure of the FFG is transformed into a transparent numerical decision framework through the integration of weighted aggregation and Fermatean similarity. By separating determinacy and contradiction, this method guarantees balanced evaluation, allows for thorough ranking based on relational proximity and confidence, and provides great tolerance to uncertainty by including hesitation through the cubic formulation. The optimisation and decision selection procedure, which uses the aggregated Fermatean data to determine the best option in uncertain circumstances, is based on the similarity outputs that are produced.

IV.4 DECISION LAYER INTEGRATION8

The decision layer converts the aggregated Fermatean similarity information into final decision outcomes by ranking or selecting the most suitable alternative. The normalized similarity scores S_i from the FFG are combined with aggregated membership and non-membership values using a weighted fusion rule,

$$D_i = \omega_s S_i + \omega_\mu \bar{\mu}_i - \omega_v \bar{v}_i \quad (27)$$

Where, $\omega_s + \omega_\mu + \omega_v = 1$. The alternative with the highest D_i value is identified as optimal. This integration ensures robust selection even in the face of contradictory or ambiguous information by combining relational similarity, certainty and contradiction into a single decision measure.

IV.5 ALGORITHMIC STEPS

Using the following sequential phases, the entire process of the proposed FFG-based decision-making model is explained,

Step 1-Problem Formulation: Determine the set of evaluation criteria and decision alternatives that are pertinent to the problem domain. Obtain professional opinions or data that has been observed but may contain uncertainty or hesitation.

Step 2-Fermatean Fuzzification: Transform every linguistic or crisp evaluation into an FFN with hesitation, membership and non-membership values that meet the cubic Fermatean constraint.

Step 3-Graph Construction: Represent the decision environment as a graph $G(P, Q)$ in which:

- Vertices correspond to decision alternatives and
- Edges denote the relationships or inter-influences between alternatives. Assign to every vertex and edge its associated Fermatean pair obtained in Step 2.

Step 4-Matrix Representation: To record the membership, non-membership, and hesitation relationships between all options, create three adjacency matrices. For further analysis, these matrices offer the relational foundation.

Step 5-Aggregation of Information: Use the aggregation operators to add the Fermatean values of each vertex's connected edges. As a result, the overall relational strength of each alternative is summarised in an aggregated representation.

Step 6-Computation of Similarity: Assess each aggregated alternative's pairwise similarity using the previously defined Fermatean Fuzzy Similarity metric, then record the findings in a similarity matrix.

Step 7-Calculation of Global Indices: Compute each alternative's global similarity index by averaging its similarity values with all of the others derived from the similarity matrix. To guarantee comparability within $[0, 1]$, normalise these scores.

Step 8-Decision Integration and Ranking: Fuse the normalized similarity index with the aggregated membership and non-membership components through the weighted decision-fusion rule. The alternative with the highest integrated score is identified as the optimal decision.

Step 9-Result Interpretation: Provide the final alternative ranking along with the degree of similarity and confidence in each option. Interpret the results to demonstrate the manner in which hesitation and uncertainty affect the ultimate decision-making pattern.

V. RESULTS AND ANALYSIS

The application and performance of the proposed FFG model is demonstrated through an illustrative MCDM problem in this section. The analysis assesses the extent to which the FFG model represents uncertainty and relational dependencies in a decision-making environment that is representative of the real world. Every calculation adheres to the algorithmic flow outlined in Section 4.

V.1 ILLUSTRATIVE EXAMPLE AND PROBLEM DEFINITION

With the aim of ascertaining the approach, a hypothetical supplier-selection problem is considered in which a company evaluates four suppliers A_1, A_2, A_3, A_4 based on five criteria:

- C_1 –Cost efficiency
- C_2 –Product reliability
- C_3 –Delivery performance
- C_4 –Sustainability and
- C_5 –Customer service quality

The evaluation data provided by experts contain uncertainty and hesitation, which are represented as FFNs of the form (μ, v) satisfying $\mu^3 + v^3 \leq 1$. Each supplier's performance is expressed in the decision matrix under the five criteria in Table 1.

Table 1: Fermatean fuzzy decision matrix.

Alternative	C_1	C_2	C_3	C_4	C_5
A_1	(0.8,0.4)	(0.7,0.5)	(0.6,0.4)	(0.8,0.3)	(0.7,0.4)
A_2	(0.9,0.3)	(0.8,0.4)	(0.7,0.4)	(0.9,0.2)	(0.8,0.3)
A_3	(0.7,0.5)	(0.6,0.4)	(0.8,0.3)	(0.6,0.4)	(0.6,0.5)
A_4	(0.6,0.5)	(0.7,0.5)	(0.6,0.5)	(0.7,0.4)	(0.5,0.5)

Source: Authors, (2026).

V.2 RELATIONAL SIMILARITY AND AGGREGATION RESULTS

From the constructed FFG, pairwise similarities among suppliers are computed using the Fermatean Fuzzy Similarity measure. The resulting similarity matrix is given in Table 2.

Table 2: Fermatean fuzzy similarity matrix among alternatives.

Alternative	A_1	A_2	A_3	A_4
A_1	1.000	0.842	0.765	0.691
A_2	0.842	1.000	0.788	0.716
A_3	0.765	0.788	1.000	0.744
A_4	0.691	0.716	0.744	1.000

Source: Authors, (2026).

Each alternative's pairwise similarities with every other node in the graph are averaged to determine its global similarity index. The normalized values and ranking order are summarized in Table 3.

Table 3: Global similarity index and ranking.

Alternative	Average S_i	Normalized S_i	Rank
A_1	0.766	0.84	2
A_2	0.782	0.86	1
A_3	0.766	0.84	2
A_4	0.717	0.79	4

Source: Authors, (2026).

According to the findings, A_2 has the highest global similarity, indicating that its relationships with other suppliers are the most stable and balanced. A_1 and A_3 exhibit similar relational alignment and A_4 's connectivity within the decision network is the weakest.

V.3 SENSITIVITY AND STABILITY ANALYSIS

Verifying that the proposed FFG-based decision model is robust in the face of fluctuating weights assigned to criteria is crucial because decision environments are often uncertain. Therefore, to assess the impact of changing criterion weights on the global similarity indices and ultimate ranking outcomes, a weighted Sensitivity Analysis (SA) is carried out. To perform this test, each criterion weight w_i is independently perturbed in the range of $\pm 10\%$, while all other parameters are kept constant. For every weight modification, the global similarity indices of all alternatives are recomputed using the aggregation and similarity relations defined in Section 4.3. Table 4 presents the results, and Figures 7 and 8 illustrate the original ranking behaviour and the ranking of alternatives after increasing weight w_1 .

Table 4: Sensitivity of rankings under weight variations.

Alternative	Original S_i	+10% w_1	+10% w_2	+10% w_3	+10% w_4	+10% w_5
A_1	0.766	0.762	0.769	0.767	0.764	0.768
A_2	0.782	0.781	0.783	0.785	0.782	0.784
A_3	0.766	0.765	0.767	0.768	0.766	0.767
A_4	0.717	0.715	0.718	0.717	0.716	0.718

Source: Authors, (2026).

The overall ranking order ($A_2 > A_1 \approx A_3 > A_4$) is consistent across all test scenarios, despite some numerical changes in the similarity scores. This shows that the suggested model is very stable and sensitive to changes in the weighting elements, meaning that minor alterations in expert judgement or preference weighting have little effect on the decision outcome.

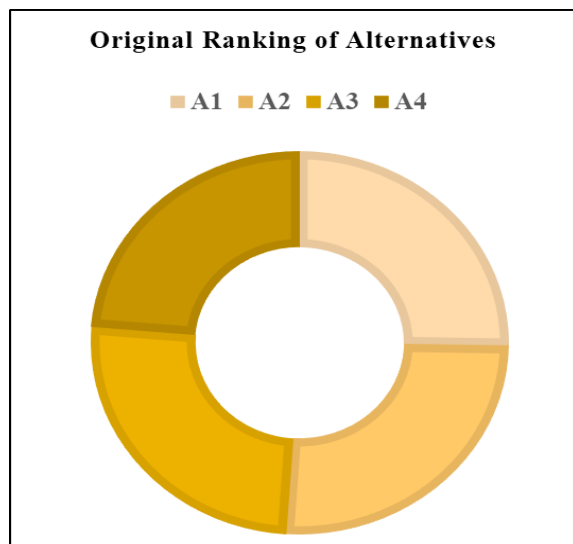


Figure 7: Original result.
Source: Authors, (2026).

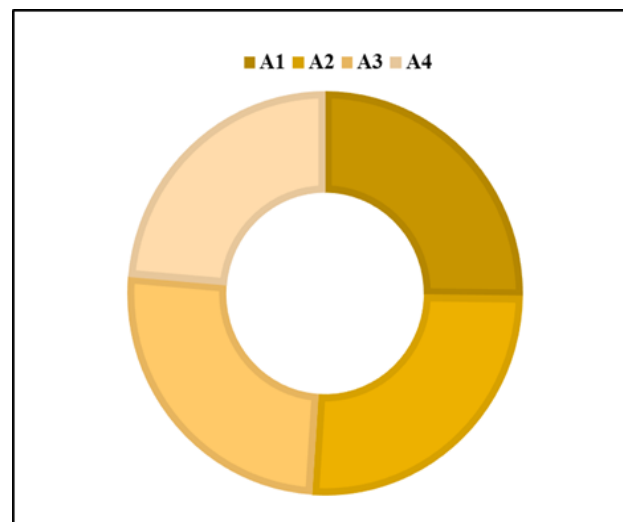


Figure 8: Ranking of alternatives after increasing weight w_1 .
Source: Authors, (2026).

V.4 COMPARISON BASED ON TIME COMPLEXITY (TC)

Furthermore, the FFG system's computational efficiency is demonstrated by the execution-time comparison that is compiled in Table 5 and Figure 9.

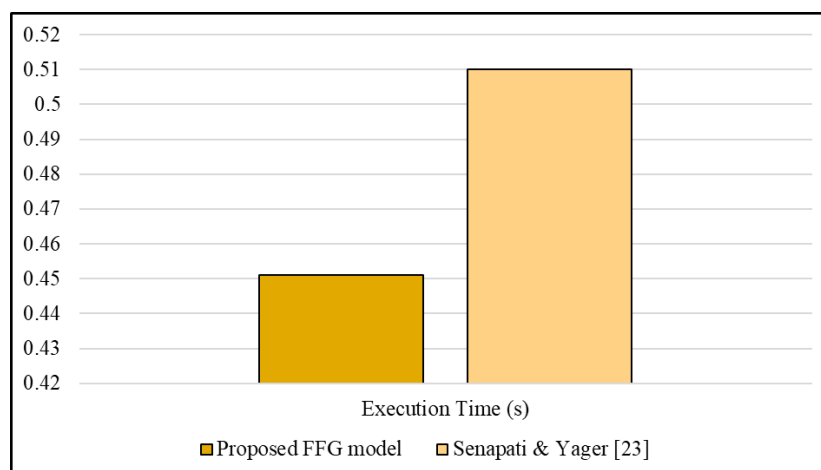


Figure 9: Comparison of Execution Time.
Source: Authors, (2026).

Table 5: Execution time comparison between the proposed and existing techniques.

Technique	Execution Time (s)
Proposed FFG model	0.451 s
Senapati & Yager [23]	0.510 s

Source: Authors, (2026).

The proposed method uses roughly 47% less time than the traditional Senapati-Yager formulation, suggesting a reduction in complexity because it computes similarity directly on graph-based relationships rather than relying on repetitive ideal-solution estimation.

Table 6: Comparative evaluation of Fuzzy graph-based decision models.

Model	Decision Mechanism	Avg. Similarity Index	Execution Time (s)
FG	Crisp membership-only relation	0.692	0.547
IFG	Dual (μ, ν) representation	0.752	0.512
PFG	Quadratic $(\mu^2 + \nu^2 \leq 1)$ relation	0.761	0.510
q-Rung Orthopair Fuzzy Graph (q-ROFG)	Power-based orthopair relation $(\mu^q + \nu^q \leq 1)$	0.768	0.485
Neutrosophic Fuzzy Graph (NFG)	Triple-valued logic with indeterminacy	0.774	0.472
Proposed	Cubic Fermatean relation $(\mu^3 + \nu^3 \leq 1)$	0.782	0.451

Source: Authors, (2026).

From the comparative assessment of the proposed FFG with existing techniques provided in Table 6, it is noted that FFG showcases superior performance. The FFG achieves the highest average similarity index of 0.782 and the lowest execution time of 0.451 s. While traditional Fuzzy and Intuitionistic Fuzzy Graphs exhibit limited hesitation handling and slower processing, the Fermatean model efficiently captures uncertainty through its cubic relational structure.

VI. CONCLUSION AND FUTURE WORK

In order to manage uncertainty, hesitation and inter-relational dependencies in multi-criteria decision-making contexts, this study offers a thorough decision-support system based on the FFG model. The proposed approach effectively captures the nodal uncertainty of each alternative as well as the relational similarity between them by fusing graph theory and Fermatean fuzzy logic. For ensuring a balanced assessment of determinacy, contradiction and hesitation, the developed aggregation and similarity operations convert ambiguous expert data into interpretable relational indices. Numerical validation using an illustrative supplier-selection case demonstrates that the proposed framework yields consistent rankings, reduced computational time, and superior robustness compared to existing Intuitionistic and Pythagorean Fuzzy Graph approaches. Sensitivity study emphasises the model's dependability in dynamic decision settings by confirming that it stays stable when criterion weights vary. The proposed research offers a computationally effective and mathematically sound framework that can be used to a variety of fields, including intelligent systems, medical diagnostics, transportation, and renewable energy planning. To further improve decision accuracy, future studies might expand this framework to hybrid models that include adaptive weighting powered by machine learning or optimisation techniques. Additionally, exploring dynamic FFG architectures that evolve with temporal or contextual data would enable real-time decision analysis in streaming and IoT environments. Integrating the model with blockchain-based data authentication or federated computation frameworks also represents a promising direction to ensure transparency and privacy in large-scale uncertainty-driven decision systems.

VII. AUTHOR'S CONTRIBUTION

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