



RESEARCH ARTICLE

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ROBUSTNESS IMPROVEMENT OF LINEAR SYSTEMS RESPONSE USING A FRACTIONALIZED ORDER PID CONTROLLER

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ABSTRACT

In recent years, research has increasingly concentrated on fractional order systems and their approximation techniques. These systems have proven valuable for improving plant dynamics in both time and frequency domains. Proportional-Integral-Derivative (PID) controllers are widely being used in industries for process control applications. The merit of using PID controllers lie in its simplicity of design and good performance including low percentage overshoot and small setting time for slow industrial processes. The performance of PID controllers can be further improved by appropriate settings of fractional-I and fractional-D actions. This paper attempts to study the behavior of fractionalized PID controllers over integer order PID controllers in order to improve the robustness of linear systems. Compared to conventional integer-order PID controllers, the fractionalized PID controller exhibits superior performance in terms of stability margins, disturbance attenuation, and adaptability to dynamic changes in the controlled process. These findings confirm that the fractionalization approach represents a promising and practically viable solution for the design of robust control systems, offering a meaningful contribution to the field of fractional-order control engineering. This approach is generalizable to other numerical approximation methods and applicable to broader control systems applications.



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I. INTRODUCTION

The generalization of well-known differentiation and integration operations to arbitrary orders is the focus of fractional calculus, often known as fractional-order (FO) differential calculus. Science started working in the field about 1695, thanks to correspondence between two mathematicians, Leibniz and L'Hospital [1]. Over the past few decades, fractional order PID has developed quickly and has many uses in control engineering and research [2–5]. In 1997, Pdlubny was the first to use FOPID [6]. The authors [7] created FOPID and FOPI controllers, which enhanced the control performance and system robustness when compared to the conventional PID tuning techniques. Fractional-order PID (FOPID) controllers have attracted growing interest over the past few years, both in academic research and industrial practice. Their primary advantage over conventional PID controllers lies in the greater design flexibility offered by their five tunable parameters rather than three. However, this additional degree of freedom comes at the cost of significantly increased tuning complexity. To address this challenge, a wide range of design methods for FOPID controllers has been proposed in the literature [8–10]. The authors of [8] used a fractionalized PID controller strategy to reduce effect noise in the conventional feedback control loop PID controller by introducing fractional order filters. The authors of [9] have demonstrated the superiority of fractionalized PID controller with Matsuda approximation over classical ones. Fractional differentiators and integrators are important in fractional filter-based signal processing and fractional feedback management of complex and chaotic systems [10]. A fractional order system in the s-domain is an infinite-dimensional filter with an irrational continuous time transfer function.

The Carlson methodology [11], Matsuda approximation method [12], Charef approximation method [13], Oustaloup approximation method [14], and the method provided by Xue et al. [15] are all well-known ways to produce the rational approximation. The advantages of fractionalized PID based on the HHO algorithm, which uses fractional order filters to approximate integer order transfers in the feedback control loop, are demonstrated in this study. The automation designer can apply the attributes and dynamics of fractional order to the rational system under consideration by inserting fractional order integrators into the classical feedback control loop without modifying the overall equivalent closed loop transfer function. The aim of this paper is to present a novel robust control strategy that incorporates fractional-order filters into a classical feedback control loop, while preserving the overall equivalent closed-loop transfer function. Given a transfer function $G(s)$ belonging to the original control loop, the proposed idea consists in "fractionalizing" it as follows:

$$G(s) = (G(s))^\alpha \cdot (G(s))^{1-\alpha} \quad (1)$$

Where α is a real number satisfying the required condition. The goal is to retain the stability properties of the original PID control scheme while enhancing its robustness against external disturbances and noise, by leveraging the distinctive properties of fractional-order systems. Although conceptually straightforward, this approach is expected to provide control design engineers with a valuable additional tool for handling plants under realistic industrial conditions. The paper is organized as follows: Section 2 presents the fundamental definitions of fractional-order systems. Section 3 describes the integer and fractionalized PID controller. Section 4 presents simulation results and discussion. Finally, Section 6 provides concluding remarks and outlines future research

II. FRACTIONAL ORDER SYSTEMS

II.1 DEFINITIONS OF FRACTIONAL CALCULUS

Fractional calculus is an old mathematical topic since 17th century. Fractional calculus is a subdivision of calculus theory which generalizes the derivative or integral of a function to non-integer order. Fractional calculus helps evaluating $(d^n y/dt^n)$, n -fold integrals where n is fractional, irrational or complex. For fractional order systems n is considered to be fractional. The number of applications where fractional calculus has been used rapidly grows. These mathematical phenomena allow to describe a real object more accurately than the classical —integer-order methods. The real objects are generally fractional however, for many of them the fractionality is very low. The main reason for using the integer-order models was the absence of solution methods for fractional differential equations. At present there are lots of methods for approximation of fractional derivative and integral and fractional calculus can be easily used in wide areas of applications (e.g.: control theory - new fractional controllers and system models, electrical circuits theory - fractances, capacitor theory, etc.) [1], [2], [8], [10]. The generalized fundamental operator which includes the differentiation and integration is given as:

$${}_a D_t^q = \begin{cases} \frac{d^q}{dt^q} & , R(q) > 0 \\ 1 & , R(q) = 0 \\ \int_a^t (d\tau)^{-q} & , R(q) < 0 \end{cases} \quad (2)$$

Where,

a: Lower limit of integration

t: Upper limit of integration

q: Order of fractional differentiation or integration q with negative value indicates integration while q with positive value indicates differentiation [3]. The theory of fractional-order derivative was developed mainly in the 19th century. There are several definitions of fractional order derivative. Two important and widely applied definitions are Grünwald-Letnikov definition is perhaps the best known due to its most suitability for the realization of discrete control algorithms [1], [15], [16]. The Grünwald-Letnikov definition is expresses as:

$${}_a D_t^\alpha f(t) = \lim_{h \rightarrow 0} h^{-\alpha} \sum_{j=0}^{\frac{(t-a)}{h}} (-1)^j \binom{\alpha}{j} f(t - jh) \quad (3)$$

Where, $[x]$ is the integer part of x and h is the step time. The Riemann-Liouville definition is expresses as:

$$f(t) = \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \quad (4)$$

For a wide class of functions which appear in real physical and engineering applications, the Riemann-Liouville and the Grünwald-Letnikov definitions are equivalent [1].

II.2 DEFINITION OF FRACTIONAL ORDER SYSTEMS

Feedback control system is one of the major areas where the concept of fractional calculus is being applied to obtain an efficient system and also longevity and freedom. To the control engineer to compensate any shifts in the transfer function due to parametric spreads, aging etc. a system is efficient if the controller is of the similar order to that of a plant being controlled. In reality the systems are of fractional order and therefore to have a fractional order controller will be efficient [17-20]. The transfer function of a fractional order system is given as:

$$G(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \dots + b_1 s^{\beta_1} + b_0}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \dots + a_1 s^{\alpha_1} + a_0} \quad (5)$$

Where a_i and b_j are real numbers such that and s is the Laplace operator.

$$\begin{cases} 0 \leq \alpha_0 \leq \alpha_1 \leq \dots \leq \alpha_n \\ 0 \leq \beta_0 \leq \beta_1 \leq \dots \leq \beta_m \end{cases}$$

III.3 CHAREF METHOD

The Charef 's transfer method given in [13], [21] is based on approximation of the function as;

$$G_f(s) = s^\alpha, \alpha \in R^+ \quad (6)$$

These processes in the frequency domain can be described by an approximation in the Laplace domain such as:

$$G_f(s) = s^\alpha \approx \left(1 + \frac{s}{P_T}\right) \approx \frac{\prod_{i=0}^N (1 + \frac{s}{P_i})}{\prod_{i=0}^{N-1} (1 + \frac{s}{Z_i})} \quad (6)$$

Where $P_i = (ab)^i \cdot P_0$, $i = 1, 2, 3, \dots, N$,

$Z_i = (ab)^i \cdot a \cdot P_0$, $i = 2, 3, \dots, N - 1$ and $P_0 = P_T \cdot 10^{\frac{ep}{20\beta}}$, $\alpha = 10^{\frac{ep}{10(1-\beta)}}$, $b = 10^{\frac{ep}{10\beta}}$, $\beta = \frac{\log(a)}{\log(a \cdot b)}$

III. FRACTIONALIZED AND INTEGER ORDER PID CONTROLLER

The feedback control loop of an integer order system is shown in figure (1) as,

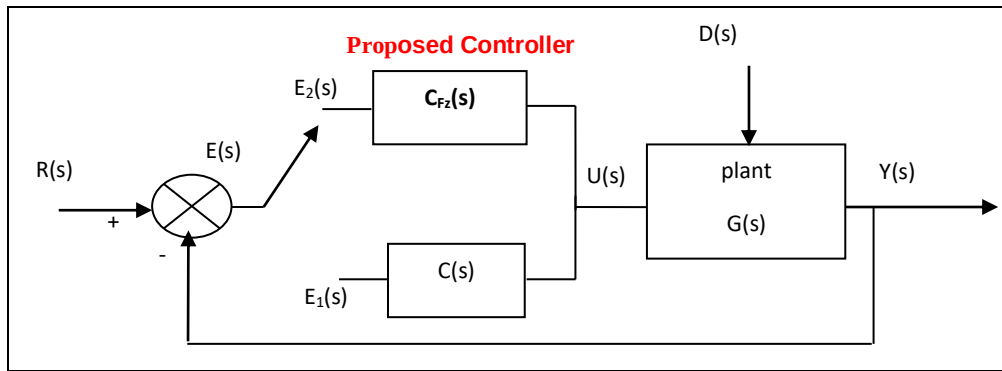


Figure 1: Fractionalized order PID Controller of the linear System.

Source: Authors, (2026).

Where, $U_R(s)$: Input Signal ,

$E(s)$: Error Signal

$C(s)$: Controller Transfer Function

$C_{FZ}(s)$: Fractionalized Controller Transfer Function

$G(s)$: System or plant Transfer Function

$Y(s)$: output Signal $U(s)$ – Controller Signal

The integer-order PID controller [22] to be designed is in the following form:

$$C(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s\right) \quad (7)$$

The Fractionalized of the integer-order PID controller to be designed is in the following form [16-20]:

$$C_{FZ}(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s\right) = \frac{1}{s} \left(\frac{(k_p T_d s^2 + k_p T_i s + k_p)}{T_i} \right) = \frac{1}{s^\alpha} \frac{1}{s^{(1-\alpha)}} \left(\frac{(k_p T_d s^2 + k_p T_i s + k_p)}{T_i} \right) \quad (8)$$

Where, $0 < \alpha < 1$

III. SIMULATION RESULTS AND DISCUSSION

The model of the system is given by the following transfer function:

$$G(s) = \frac{(s + 3)}{(s^3 + 5s^2 + 2s + 1)} \quad (9)$$

A PID controller is design for the integer order model of the system for the following parameters for the PID controller: $K_p = 15$, $K_i = 0.5$, $K_d = 1.8$ Figure (2) shows the time response characteristics for a step change of the system (9) Table 1 shows its time response specifications.

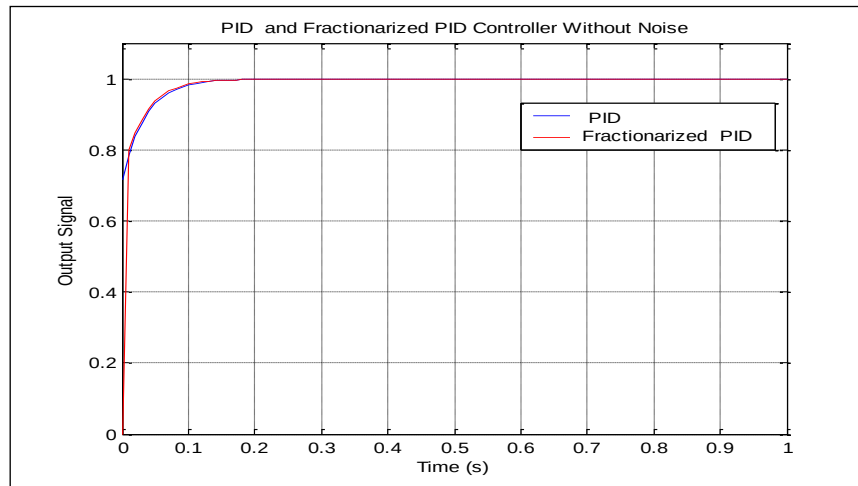


Figure 2: PID and fractionalized order PID controllers are designed for the system control process Without Noise.
Source: Authors, (2026).

Figure (3) shows the time response characteristics for a step change of the system with random output noise of 10 % of the reference signal amplitude ($\alpha=0.3$):

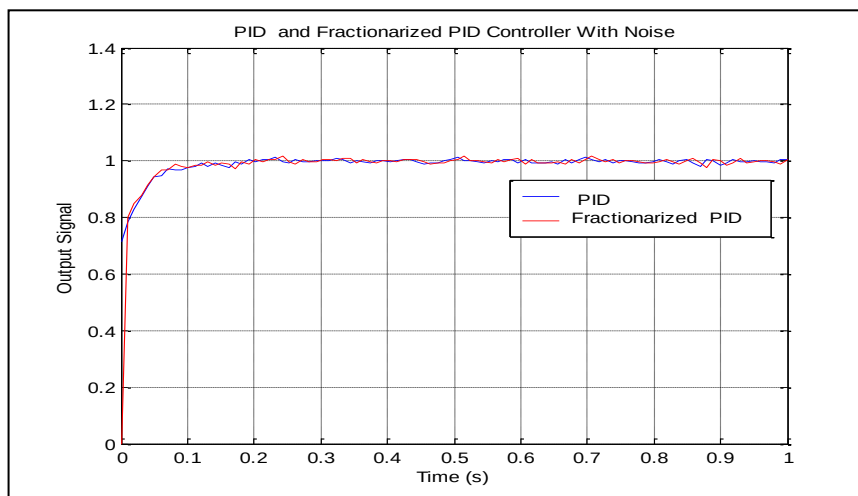


Figure 3: PID and fractionalized order PID control process with random output noise of 10 % of the reference signal amplitude ($\alpha=0.3$).

Source: Authors, (2026).

Figure (4) shows the time response characteristics for a step change of the system with random output noise of 25 % of the reference signal amplitude ($\alpha=0.3$)

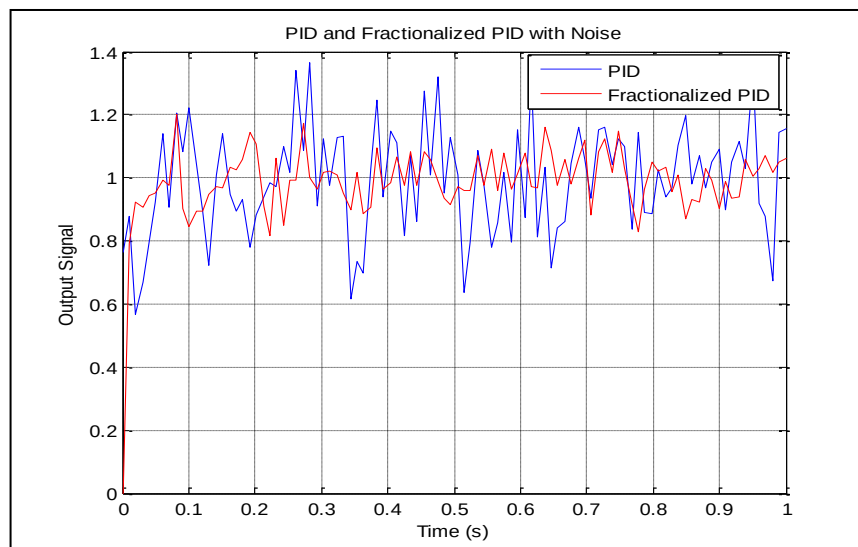


Figure 4: PID and fractionalized order PID control process with random output noise of 25 % of the reference signal amplitude ($\alpha=0.3$).

Source: Authors, (2026).

The evaluation of the control system performance will be realized by dening a quadratic error criterion J_α given by [23]:

$$J_\alpha = \int_{t_I}^{t_F} (U_R(t) - Y(t))^2 dt \quad (10)$$

The table 1 show the Quadratic error criterion with random output noise of 5-25 % .

Table 1: Quadratic error criterion with random output noise of 5-25%.

α	0.1	0.2	0.3	0.4	0.5	1
with random output noise of 5%	0.07	0.08	0.085	0.065	0.075	0.15
with random output noise of 10%	0.15	0.17	0.16	0.13	0.15	0.29
with random output noise of 15 %	0.23	0.22	0.24	0.25	0.23	0.46
J_α with random output noise of 25%	0.31	0.34	0.32	0.33	0.35	0.75

Source: Authors, (2026).

We remark that the fractionalized PID give a certain dimunition of the noise effect of about 50 % comparatively to the classical PID result when random output noise is 5 to 25 % (Table1).The findings derived from this study clearly indicate that the fractionalization approach yields a significant improvement in the noise rejection performance of the proposed control scheme, rendering it markedly more robust against disturbances and measurement uncertainties that are typically encountered in practical industrial environments. In addition, the introduced methodology demonstrates a considerable strengthening of the overall system robustness, especially in the presence of plant parameter perturbations and modeling inaccuracies that inevitably arise in real-world control applications. When benchmarked against traditional integer-order PID controllers, the fractionalized PID controller consistently achieves superior control performance, particularly in terms of enhanced stability margins, more effective disturbance attenuation, and greater adaptability to dynamic variations within the controlled process. The simulation and analytical results collectively validate the effectiveness and reliability of the fractionalization strategy, confirming its potential as a highly promising and practically feasible framework for the development of advanced robust control systems. As such, this work constitutes a valuable and meaningful contribution to the growing body of knowledge in the field of fractional-order control engineering, opening new perspectives for the design of high-performance controllers in complex and uncertain dynamical systems.

IV. CONCLUSIONS

In this work, a novel approach to robust PID control has been proposed, based on the systematic introduction of fractional-order filters into the classical feedback control loop. A key advantage of this methodology lies in the fact that the integration of fractional-order operators does not alter the overall equivalent closed-loop transfer function, thereby preserving the fundamental stability and structural integrity of the original control system. The core idea behind the proposed fractionalization approach is conceptually straightforward yet highly effective: by embedding fractional-order integrators and differentiators into the conventional control loop, it becomes possible to harness the inherent properties of fractional-order calculus such as memory effects, non-integer dynamics, and enhanced frequency without requiring a complete redesign of the existing control architecture. The results obtained throughout this study demonstrate that the fractionalization approach significantly improves the noise rejection capability of the control scheme, making it considerably more resilient to disturbances and measurement uncertainties that are commonly encountered in real-world industrial applications. Furthermore, the proposed method leads to a substantial enhancement in the robustness of the overall control system, particularly with respect to plant parameter variations and modeling uncertainties. Compared to conventional integer-order PID controllers, the fractionalized PID controller exhibits superior performance in terms of stability margins, disturbance attenuation, and adaptability to dynamic changes in the controlled process. These findings confirm that the fractionalization approach represents a promising and practically viable solution for the design of robust control systems, offering a meaningful contribution to the field of fractional-order control engineering. Future research directions may include the extension of this approach to nonlinear and time-varying systems, as well as the investigation of alternative fractional-order approximation methods to further optimize control performance under varying operating conditions.

V. AUTHOR'S CONTRIBUTION

Conceptualization: Zahir Asradj, Bensafia Yassine.

Methodology: Zahir Asradj, Bensafia Yassine.

Investigation: Zahir Asradj, Bensafia Yassine.

Discussion of results: Zahir Asradj, Bensafia Yassine.

Writing – Original Draft: Bensafia Yassine.

Writing – Review and Editing: Zahir Asradj, Bensafia Yassine.

Resources: Zahir Asradj, Bensafia Yassine.

Supervision: Zahir Asradj, Bensafia Yassine.

Approval of the final text: Zahir Asradj, Bensafia Yassine.

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