

MOAHHGSO-W: A NEW HYBRID ALGORITHM FOR SOLVING MULTI-OBJECTIVE OPTIMIZATION PROBLEMS

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ABSTRACT

To effectively address multi-objective optimization problems, this study proposes a novel hybrid optimization approach, referred to as MOAHHGSO-w. This method integrates the characteristics of the multi-objective Weighted Sum (WS) algorithm with two recently developed single-objective optimization techniques: the Artificial Hummingbird Algorithm (AHA) and Henry Gas Solubility Optimization (HGSO). Furthermore, MOAHHGSO-w utilizes three distinct population archives to store the non-dominated solutions identified during the search process. To enhance solution diversity, the algorithm incorporates food source selection and leader selection mechanisms, both of which rely on the crowding distance metric. To achieve an optimal balance between solution diversity and convergence toward the Pareto-optimal set, the proposed method employs a combination of crowding distance and epsilon dominance. The algorithm is evaluated on four real-world multi-objective engineering design challenges. Its performance is benchmarked against several well-established multi-objective optimization methods. To validate the experimental findings, the study employs two widely recognized performance metrics: Inverted Generational Distance (IGD) for assessing convergence and Spacing (Sp) for evaluating solution diversity. The statistical analysis confirms that the proposed MOAHHGSO-w algorithm outperforms the compared optimization techniques in terms of efficiency and effectiveness.



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1. INTRODUCTION

Multi-objective optimization (MOO) is a powerful paradigm in the field of optimization that addresses scenarios where decision-making involves conflicting objectives. Unlike regular optimization problems that focus on one goal, multi-objective optimization (MOO) aims to improve several competing goals at the same time [1]. In various real-world applications, decision-makers often need to balance multiple goals, each contributing to the overall performance of a system. These conflicting objectives can range from minimizing costs and maximizing efficiency to achieving a balance between multiple performance metrics [2]. MOO provides a systematic framework to address the complicated trade-offs inherent in such decision-making processes. A fundamental aspect of multi-objective optimization is the identification and exploration of Pareto-optimal solutions [1]. A solution is considered Pareto-optimal if no other feasible solution improves one objective without worsening at least one other objective. The set of all Pareto-optimal solutions forms the Pareto front, representing the trade-off curve between conflicting objectives [3]. Solution approaches for multi-objective optimization problems (MOOPs) can be broadly categorized into classical, evolutionary, and hybrid methods. Classical techniques, such as the weighted sum and epsilon-constraint methods, transform MOOPs into single-objective problems [4]. Although computationally efficient, these methods often struggle with non-convex or complex Pareto fronts. To overcome these limitations, evolutionary algorithms have been widely adopted due to their population-based nature and ability to approximate multiple Pareto-optimal solutions in a single run. Prominent multi-objective evolutionary algorithms include NSGA-II [5], SPEA2 [6], and MOEA/D [7-9].

At the same time, nature-inspired metaheuristic algorithms like MOPSO [10], MOFO [11], MOAWA [12], and MOCSO [13] have shown that they can balance convergence and diversity very well. Recent advances further include MOAOA [14], MOMRFO [15-17], MOHHO [18-20], and other enhanced swarm-based techniques [16], [21-25]. Hybrid metaheuristic algorithms have emerged as powerful tools for solving multi-objective optimization problems (MOO) by integrating different optimization techniques to enhance solution quality, convergence speed, and robustness. Notable hybrid approaches include the multi-objective gravitational search algorithm and the bat algorithm (MOGSABAT) [2]. The MoSSE algorithm [26] is a hybrid optimization method that combines the features of Multi-Objective Spotted Hyena Optimizer (MOSHO), Salp Swarm Algorithm (SSA), and Emperor Penguin Optimizer (EPO). The firefly algorithm [27] is hybridized with a genetic algorithm for multi-objective integrated process planning and scheduling, while another approach is the metaheuristic hybrid of the multi-objective ant lion optimizer and genetic algorithm designed for solving design problems (MOALG) [28]. There is no perfect algorithm for solving all multi-objective optimization problems, as the choice depends on the nature of the problem, its constraints, and the desired objectives [29]. This is what motivates and encourages researchers to develop novel optimization techniques. Given this motivation, we propose a new multi-objective optimization method. Motivated by these observations, this paper proposes a novel hybrid multi-objective optimization algorithm, termed MOAHHGSO-w, which combines the Artificial Hummingbird Algorithm (AHA) and Henry Gas Solubility Optimization (HGSO), extended to the multi-objective domain using a weighted sum strategy. The main contributions of this work are summarized as follows:

- A new hybrid framework integrating AHA, HGSO, and a weighted sum approach for solving multi-objective optimization problems.
- A multi-archive mechanism, including local, global, and temporal archives, to effectively preserve non-dominated solutions.
- A leader selection and food source targeting strategy based on crowding distance to enhance population diversity.
- An epsilon-dominance-based updating mechanism to improve Pareto front distribution and convergence.
- Extensive validation using UF and DTLZ benchmark problems, as well as real-world engineering applications.

The remainder of this paper is organized as follows: Section 2 reviews fundamental concepts of multi-objective optimization. Section 3 details the proposed methodology, while Section 4 presents the MOAHHGSO-w algorithm. Section 5 discusses experimental results and performance analysis, and Section 6 concludes the paper.

II. THEORETICAL REFERENCE

II.1 MULTI-OBJECTIVE OPTIMIZATION PROBLEM

A general multi-objective optimization problem (MOOP) can be formulated as follows:

$$\begin{array}{ll}
 \text{Min / Max} & OF(V) = [of_1(V), of_2(V), \dots, of_M(V)] \\
 \text{Subject to} & C_i(V) \leq 0 \quad i = 1, 2, \dots, I \\
 & H_e(V) = 0 \quad e = 1, 2, \dots, E \\
 & V_j^L \leq V_j \leq V_j^U \quad i = 1, 2, 3, \dots, n
 \end{array} \quad (1)$$

Where V is an n -dimensional decision vector, M denotes the number of objectives .functions, and V_j^L , V_j^U are the lower and upper bounds of decision variables, respectively.

Definition 1: Let $V_1, V_2 \in \mathbb{R}^M$, we say that solution V_1 dominates solution V_2 if:

- Weak dominance: $\forall i, i = 1, 2, \dots, M, of_i(V_1) \leq of_i(V_2)$, and
- Strong dominance $\exists i, i = 1, 2, \dots, M, of_i(V_1) < of_i(V_2)$.

Definition 2: Solutions that dominate others but do not dominate one another are referred to as a non-dominated solutions set. The set of non-dominated solutions within the complete search space is referred to as the Pareto-optimal set.

Definition 3: Let V_i and V_j be two solutions in the objective space, where each solution is represented by a vector of objective values. A solution V_i is said to ε -dominate a solution V_j if the following conditions are met:

$$(1 + \varepsilon_m)of_m(V_i) \leq of_m(V_j) \quad \forall m \in \{1, \dots, M\} \quad (2)$$

Where OF_m is the value of objective m for solution V , ε_m is the allowed margin for objective m , and M is the total number of objectives.

Definition 4: Crowding distance is a density estimation technique used to preserve diversity among non-dominated solutions [5].

II.2 PERFORMANCE METRICS

To evaluate the quality of solutions generated by multi-objective optimization algorithms, several performance metrics are adopted.

II.2.1 Generational Distance (GD)

The Generational Distance metric measures the convergence of obtained solutions toward the true Pareto front [30]. It is defined as:

$$GD = \frac{1}{|V|} \left(\sum_{i=1}^{|V|} D_i^p \right)^{\frac{1}{p}} \quad (3)$$

Where D_i is the Euclidean distance from solution v_i to the nearest solution in the reference Pareto front P , p is typically set to 2 for Euclidean distance. Smaller GD values indicate better convergence.

II.2.2 Inverted Generational Distance (IGD)

Unlike Generational Distance (GD), which measures the convergence of solutions to the Pareto front, Inverted Generational Distance (IGD) also considers diversity by assessing how well the obtained solutions cover the entire optimal front.

$$IGD = \frac{1}{|m|} \left(\sum_{j=1}^{|m|} D_j'^p \right)^{\frac{1}{p}} \quad (4)$$

Where D_j is the Euclidean distance from each reference point p_j in the Pareto front to the nearest solution in the obtained set V , p is typically set to 2 for Euclidean distance. Lower IGD values indicate better coverage and distribution.

II.2.3 The Spacing Metric

The spacing metric evaluates the uniformity of solution distribution along the Pareto front:

$$Sp = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (D_i - \bar{D})^2} \quad (5)$$

Where: D_i is the minimum Euclidean distance from solution v_i to any other solution v_j in the obtained set ($i \neq j$), $\bar{D}$ is the mean of all values. Smaller values indicate more uniform spacing.

III. MATERIALS AND METHODS

III.1 ARTIFICIAL HUMMINGBIRD ALGORITHM

The Artificial Hummingbird Algorithm (AHA), proposed by Zhao [31], is a nature-inspired metaheuristic based on the intelligent foraging behavior, flight patterns, and memory mechanisms of hummingbirds. These characteristics enable AHA to efficiently balance exploration and exploitation in complex, high-dimensional optimization problems. AHA employs three main foraging strategies:

- *Guided foraging*: which directs individuals toward promising and less-visited food sources:

$$v_i(t+1) = x_{i,tar}(t) + a \cdot D(x_i(t) - x_{i,tar}(t)) \quad (6)$$

where $a \sim N(0,1)$ and D is a direction-switching vector. The position update is conditional:

$$x_i(t+1) = \begin{cases} v_i(t+1) & OF(x_i(t)) > OF(v_i(t+1)) \\ x_i(t) & otherwise \end{cases} \quad (7)$$

- *Territorial foraging*: which performs local exploitation around the current position:

$$v_i(t+1) = x_i(t) + b \cdot D \cdot x_i(t); b \sim N(0,1). \quad (8)$$

- *Migrating foraging*: which randomly redistributes individuals to avoid stagnation:

$$x_{wor}(t+1) = Low + r(Up - Low) \quad x_{i,tar} \quad (9)$$

Where Low and Up define the search bounds, and $r \in [0,1]$

Additionally, AHA integrates multiple flight patterns (axial, diagonal, and omnidirectional) and a visit-table memory mechanism to enhance diversity and prevent repetitive searches. While AHA exhibits strong exploration capability, its convergence speed may be limited in complex landscape.

III.2 HENRY GAS SOLUBILITY OPTIMIZATION

The Henry Gas Solubility Optimization algorithm (HGSO), introduced by Hashim [32], is inspired by Henry's law, which governs gas solubility in liquids under varying pressure and temperature conditions. HGSO models optimization agents as gas particles interacting within clusters, enabling an effective balance between exploration and exploitation. The population is initialized randomly as

$$V_i(t+1) = V_{min} + r(V_{max} - X_{min}) \quad (10)$$

Henry's constant is dynamically updated according to

$$H_i(t+1) = H_j(t) \cdot \exp\left(-C_j \left(\frac{1}{T(t)} - \frac{1}{T_\theta}\right)\right), T(t) = \exp\left(\frac{-t}{Iter}\right) \quad (11)$$

Where $T^\theta = 298.15K$ is the reference temperature. The solubility of each agent is computed by:

$$S_{i,j}(t) = K H_j(t+1) P_{i,j}(t) \quad (12)$$

And the position update is given by:

$$V_{i,j}(t+1) = V_{i,j}(t) + Fr\delta(V_{i,best}(t) - V_{i,j}(t) + Fr\alpha(S_{i,j}(t) \cdot V_{best}(t) - V_{i,j}(t))) \quad (13)$$

$$\delta = \beta \cdot \exp\left(\frac{-OF_{best}(t) + \varepsilon}{OF_{i,j}(t) + \varepsilon}\right), \varepsilon = 0.05$$

Parameters δ, α and F ensure diversity and effective navigation through the search space. HGSO demonstrates good convergence properties; however, its performance may be sensitive to parameter settings and computationally demanding in high-dimensional problems.

III.3 WEIGHTED SUM METHOD

The Weighted Sum (WS) method is a classical scalarization technique used to transform multi-objective problems into single-objective formulations [4]. A composite objective function is constructed as:

$$g_{ws} = \sum_{i=1}^m w_i OF_i(x) \quad (14)$$

Where w_i represents the relative importance of the i -th objective. The WS method is computationally efficient and simple to implement; however, it may fail to capture non-convex regions of the Pareto front and is sensitive to weight selection.

III.4 MOTIVATION FOR HYBRIDIZATION

Each of the aforementioned algorithms exhibits complementary strengths and limitations. AHA offers strong exploration capability, HGSO provides effective convergence behavior, and WS facilitates multi-objective extension with low computational cost. Motivated by the No Free Lunch theorem [29], this work integrates these methods into a unified hybrid framework to enhance convergence, diversity preservation, and robustness in multi-objective optimization.

III.5 PROPOSED ALGORITHM

We present a new algorithm, MOAHHGSO-w, in this research to solve optimization problems with multiple objectives. This method combines important features from two new single-objective optimization algorithms with a well-known multi-objective method. It specifically includes parts of the Artificial Hummingbird Algorithm (AHA) [31], the Henry Gas Solubility Optimization Algorithm (HGSO) [32], and the Weighted Sum Method [4]. The MOAHHGSO-w uses the epsilon dominance relationship to compare possible solutions. In addition, the algorithm makes use of three different external archives: Local Archives, Temporary Archives, and the Global Archive. We will explain each of these archives' specific roles in the optimization process later. Once the optimization process concludes, the Global Archive provides the final set of Pareto-optimal solutions. The general structure of MOAHHGSO-w is outlined in Algorithm 1, and the subsequent section provides an in-depth explanation of its operational mechanisms.

Initialize population and parameters: The initial phase of the proposed hybrid algorithm, MOAHHGSO-w, involves generating a random set of N agents to form the initial population (Pop). Each agent is then evaluated based on M objective functions. Subsequently, the global archive (GArch) is initialized by storing the set of non-dominated solutions.

Position Update Using the AHA Algorithm: At this stage, a modified version of the Artificial Hummingbird Algorithm (AHA) is applied with additional enhancements:

- **Selecting the Target Food Source:** Each hummingbird identifies the optimal food source (solution) by referring to the visit table and selecting the one with the highest visit level. The identified food sources are then stored in a temporary archive (TArch), ensuring that dominance relationships are considered during the position update process. The MOAHHGSO-w algorithm determines the target food source for each hummingbird $X_{i,t}(t)$ at time t by retrieving the most suitable candidate from the temporary archive (ArchiveT). The following section details the specific selection mechanism employed in this process.
 - Compute the crowding distance for every food source in the temporal archive (TArch).
 - Sort the food sources in the temporary archive based on their congestion levels.
 - To enhance solution distribution along the Pareto front, a target food source is randomly selected from the top 30% of the sorted temporary archive (TArch). This strategy directs the hummingbird towards regions with lower solution density, improving diversity.

As a result, the updated position of the i th food source is determined accordingly

$$X_{i,new}(t+1) = X_{i,t}(t) + a \cdot D(X_i(t) - X_{i,t}(t)) ; a \sim N(0,1). \quad (15)$$

$$X_i(t+1) = \begin{cases} X_{i,new}(t+1) & \text{if } X_{i,new}(t) \in \text{Global Archive} \\ X_i(t) & \text{otherwise} \end{cases} \quad (16)$$

$X_i(t)$ denotes the position of the i th food source at time t . Similarly, $X_{i,t}(t)$ corresponds to the location of the target food source that the i th hummingbird intends to visit. The guided factor follows a normal distribution $N(0,1)$.

Algorithm 1: Pseudo code of the MOAHHGSO-w algorithm.

Initialization: Initialize a population of N agents, Evaluation of the initial population, and Initialize Local Archives and the Global Archive.

for $It < MaxIt$ **do**

if $rand < 0.5$ **then**

 Update position using the AHA algorithm.

 Update the global archive.

else

 Update position using the HGSO and the WSA algorithms.

 Update Locale Archives (LAch).

 Update the Global Archive (GAch).

end if

if $\text{mod}(2N, It) == 0$ **then**

 Update the position of the worst agents.

 Update the Global Archive (GAch).

end if

end for

Return the Global Archive (GAch)

Update the global archive: The MOAHHGSO-w algorithm integrates an advanced update mechanism to enhance both the diversity of solutions along the Pareto front and the convergence toward optimal solutions. To preserve non-dominated solutions at each iteration t , a restricted-size external global archive (GArch) is employed, where the maximum archive size is denoted as sizeGArch . Following each update of a hummingbirds position V_i , the archive population is refined using a structured update strategy. The epsilon dominance criterion is applied to determine whether a newly generated solution should be accepted or discarded. Additionally, the crowding distance method is utilized to measure the relative spacing among solutions, allowing them to be ranked in descending order based on their crowding distance values. To maintain the archive size within the defined limit, solutions with lower crowding density are preserved, whereas those in more congested regions are removed. Each solution within the archive population V_{GA} , along with any newly generated solution NS considered for inclusion, is assigned an identification vector $I = (I_1, I_2, \dots, I_M)$, where M represents the total number of objective functions. The identification process follows a structured approach, which will be elaborated in the subsequent section.

$$I_i(of) = \left\lfloor \frac{\log of_j}{\log(1 - \epsilon)} \right\rfloor \quad (17)$$

Where $\lfloor . \rfloor$ is the absolute value, of_j : the objective value j th of an archive solution and denotes the admissible error. The identification vector partitions the whole objective space into hyper-boxes. Due to the constraint that only one solution may be accommodated in each hyper-box on the Pareto solutions set, the solution archive exhibits a moderate level of variability. MOAHHGSO-w utilizes the crowding distance as a means to restrict the size of the global archive if it is above the specified maximum limit (sizeArch). Initially, we compute the crowding distance for each solution in the global archive, and subsequently, we order the global archive solutions based on their crowding distance to diminish the size of the archive. The solutions with a lower population density are preserved, while those with a higher population density are removed from the archive.

Update position using HGSO and WSA Algorithm: During this phase, we implement the Henry gas solubility optimization algorithm with a few alterations and enhancements:

- **Leaders selection strategy:** After dividing the population into C clusters, each cluster is linked to a corresponding local external archive (LA_j , $j=1...C$), which stores the most promising solutions for gases during the optimization process. Additionally, the main population is connected to a global archive (GA), ensuring that the best solutions within both the clusters and the overall population are preserved. This mechanism allows for maintaining high-quality solutions that have not yet been surpassed. The local archives are systematically ranked in descending order based on their crowding distance values. Subsequently, the subset of solutions with the lowest density, corresponding to the top 30% of non-dominated solutions in the sorted archives, is identified. Within this subset, a solution is randomly selected to serve as the regional leader for each cluster. This leader plays a crucial role in guiding the main population of gases toward an accurate approximation of the true Pareto-optimal solutions. A similar approach is employed to determine a global leader for each gas from the global archive (GA). This leadership selection method enhances both the convergence rate and the diversity of solutions. Assigning two leaders to each search agent further strengthens the balance between convergence and solution diversity within the population. The new formula for updating the position is:

$$V_{i,j}(t+1) = V_{i,j}(t) + F r \delta ((L_{LA}(i,j)(t) - V_{i,j}(t) + F r \alpha (S_{i,j}(t).L_{GA}(t) - V_{i,j}(t))) \quad (15)$$

$$\delta = \beta \cdot \exp\left(\frac{-SW(L_{GA}, t) + \varepsilon}{SW(V_{i,j}(t)) + \varepsilon}\right), \varepsilon = 0.05$$

We denote the position of gas i within cluster j as $V_{i,j}$, where t represents the iteration step and r is a fixed constant. The term $L_{LA}(i,j)$ refers to the leader gas i within cluster j , while L_{GA} designates the leader gas across the entire population. Additionally, δ represents the capacity of gas j within cluster i to interact with other gases in the same cluster. The parameter α , which is set to 1, indicates the influence of surrounding gases on gas i within cluster j , whereas β is considered a constant. Both $SW(L_{GA}, t)$ and $SW(V_{i,j}(t))$ correspond to the weighted sum approach.

Update local archives: The local archive update process follows the same methodology as the global archive update procedure.

Update the position of the worst agents: To refine the search process, all inferior solutions (dominated solutions) are removed and replaced with newly generated solutions that are randomly distributed throughout the entire search space. This replacement occurs every $2N$ iterations, where N denotes the population size. After each replacement, the global archive is subsequently updated. This step is very important for lowering the chance of getting stuck in local optima and making the algorithm better at exploring in general, especially as the optimization process goes on.

IV. RESULTS AND DISCUSSIONS

IV.1 THE IMPACT OF HGSO AND AHA ALGORITHMS ON THE EFFICACY OF THE PROPOSED ALGORITHM

A comparison is made between the suggested algorithm, the proposed algorithm without AHA, and the proposed algorithm without HGSO, using the IGD and SP metrics to see how HGSO and AHA affect the performance of the MOAHHGSO-w algorithm on three engineering design problems:] the four-bar truss (4-b truss) [33], the disk brake (Disk) [34], and welded beam design problem (WBD) [34]. Table 1 shows the statistical findings for the IGD metric over 31 runs of engineering design problems, while Table 2 presents the statistical results for the SP metric over the same 31 runs. These statistical results of the two metrics prove that in all engineering design problems, the proposed hybrid algorithm outperforms the other two algorithms (without AHA, without HGSO).

Table 1: IGD metric results for real engineering multi-objective design problems.

Bar	Average	STD	Disk	Average	STD	WBD	Average	STD
MOAHHGSO-w without HGSO	1.75E-01	1.08E-02	MOAHHGSO-w without HGSO	2.89E-03	1.96E-04	MOAHHGSO-w without HGSO	1.10E-02	3.17E-03
MOAHHGSO-w without AHA	3.83E-01	1.02E-01	MOAHHGSO-w without AHA	5.23E-03	5.82E-04	MOAHHGSO-w without AHA	1.17E-02	1.33E-03
MOAHHGSO-w	1.63E-01	8.78E-03	MOAHHGSO-w	2.71E-03	1.43E-04	MOAHHGSO-w	8.81E-03	4.72E-04

Source: Authors, (2026).

Table 2: SP metric results for real engineering multi-objective design problems.

Bar	Average	STD	Disk	Average	STD	WBD	Average	STD
MOAHHGSO-w without HGSO	2.45E+00	2.35E-01	MOAHHGSO-w without HGSO	8.45E-02	6.49E-03	MOAHHGSO-w without HGSO	1.57E-01	2.72E-02
MOAHHGSO-w without AHA	2.44E+14	2.06E+14	MOAHHGSO-w without AHA	1.15E-01	2.13E-02	MOAHHGSO-w without AHA	1.84E-01	3.46E-02
MOAHHGSO-w	2.42E+00	1.48E-01	MOAHHGSO-w	8.08E-02	5.99E-03	MOAHHGSO-w	1.41E-01	2.51E-02

Source: Authors, (2026).

IV.2 RESULTS ON REAL WORLD ENGINEERING MULTI-OBJECTIVE DESIGN PROBLEMS

This section evaluates the performance of the proposed MOAHHGSO-w algorithm using a set of real-world engineering optimization applications: the four-bar truss (4-b truss) [33], the disk brake (Disk) [34], the speed reducer (SRD) [33], and the welded beam design problem (WBD) [34] characterized by conflicting objectives and complex constraints. The literature widely adopts the selected problems, which represent practical industrial scenarios, to evaluate the effectiveness of multi-objective optimization algorithms. The MOAHHGSO-w algorithm was implemented using MATLAB R2015a and evaluated against five well-established multi-objective optimization algorithms: the Multi-Objective Evolutionary Algorithm Based on Decomposition (MOEA/D) [7], Multi-Objective Grey Wolf Optimizer (MOGWO) [35], Multi-Objective Harris Hawks Optimization (MOHHO) [36], Multi-Objective Salp Swarm Algorithm (MSSA) [37], and Multi-Objective Particle Swarm Optimization (MOPSO) [10]. These algorithms have been widely applied in solving multi-objective optimization problems (MOPs) across various domains due to their effectiveness in approximating Pareto-optimal solutions. For a fair and unbiased comparative analysis, all algorithms were executed under identical experimental conditions. Specifically, the number of iterations was set to 1000 to ensure sufficient exploration and exploitation of the search space, the population size was fixed at 100 to maintain consistency across algorithms, and the global archive size was maintained at 100 to store non-dominated solutions.

Furthermore, each real-world engineering application was run 31 times to consider random changes and lessen the impact of chance, which improves the trustworthiness of the results. The parameter settings were adopted as specified in their respective original studies to ensure a valid and reproducible comparison. To ensure a rigorous and objective evaluation of algorithmic performance, two widely recognized performance metrics were employed: The Inverted Generational Distance (IGD) and the Spacing (Sp) metric. The IGD metric quantifies convergence by measuring the proximity of the obtained Pareto front to the true Pareto-optimal front, whereas the Sp metric assesses diversity by evaluating the distribution uniformity of the obtained solutions. A lower IGD value indicates better convergence towards the optimal front, while a lower Sp value suggests a more evenly distributed set of solutions, both of which are desirable properties in multi-objective optimization. Tables 3 and 4 provide the results, offering a comprehensive assessment of the algorithm's efficacy across various engineering design scenarios. Table 3 indicates that, across all assessed engineering design difficulties, the MOAHHGSO-w technique consistently yields the lowest mean IGD values. The IGD metric assesses the proximity of the obtained solutions to the true Pareto front, with lower values signifying superior convergence. The results indicate that MOAHHGSO-w has better convergence than other methods, which means it provides a more accurate estimate of the best trade-off solutions. The graphical representations in Figures 1, 2, 3, and 4, illustrating the algorithm's ability to generate solutions that nearly align with the actual Pareto front, further substantiate this pronounced convergence characteristic.

Table 4 presents the statistical analysis of the SP measure, which assesses the homogeneity and diversity of the Pareto front solutions. The results show that in the disk brake and welded beam design issues, where it generates a well-distributed and uniformly spaced collection of solutions, MOAHHGSO-w works very well. The hybrid nature of the strategy enables it to effectively balance exploration and exploitation, resulting in a more diverse and uniformly distributed Pareto front in these specific difficulties. MOAHHGSO-w has worse performance compared to MOGWO and MOHHO, which demonstrate enhanced solution diversity and dispersion in the four-bar truss and speed reducer design problems. The elevated SP values for MOAHHGSO-w in these contexts indicate that the solutions generated by this method are less uniformly distributed compared to those produced by MOGWO and MOHHO. While MOAHHGSO-w maintains robust convergence characteristics, its solution diversity is contingent upon the specific problem, and in certain instances, alternative methodologies may produce more uniformly distributed Pareto fronts. The visual data from Figures 1, 2, 3, and 4 corroborates these conclusions.

Table 3: The IGD findings derived from the use of the MOAHHGSO-w, MOGWO, MOPSO, MOEA/D, MSSA, and MOHHO algorithms in engineering design challenges.

Bar	MOAHHGSO-w	MOEA/D	MOGWO	MSSA	MOPSO	MOHHO
Average	1.63E-01	1.67E+01	3.52E-01	1.02E+00	4.78E-01	1.02E+01
STD	8.78E-03	2.52E+00	8.67E-02	3.31E-01	2.21E-01	4.12E+00
Disk						
Average	2.71E-03	5.46E-02	6.63E-03	2.34E-02	8.78E-03	1.61E-01
STD	1.43E-04	2.87E-02	1.56E-03	1.04E-02	5.74E-03	3.56E-02
SRD						
Average	1.01E+01	1.20E+09	1.15E+01	1.62E+01	1.26E+01	4.78E+01
STD	7.33E-01	4.32E+00	7.23E-01	2.79E+00	3.75E+00	8.10E+00
WBM						
Average	8.80E-03	8.35E-01	4.47E-02	9.88E-02	2.04E-02	8.68E-01
STD	4.72E-04	7.03E-02	2.17E-02	7.85E-02	5.13E-03	4.12E-02

Source: Authors, (2026).

Table 4: The SP findings derived from the use of the MOAHHGSO-w, MOGWO, MOPSO, MOEA/D, MSSA, and MOHHO algorithms in engineering design challenges.

Bar	MOAHHGSO-w	MOEA/D	MOGWO	MSSA	MOPSO	MOHHO
Average	2.42E+00	7.04E-01	8.07E-01	3.38E+00	8.05E-01	5.69E-01
STD	1.48E-01	1.18E-02	1.54E-03	1.21E+00	2.88E-03	7.66E-02
Disk						
Average	8.08E-02	2.09E-01	2.27E-01	1.40E-01	2.34E-01	2.01E-01
STD	5.99E-03	1.43E-02	2.90E-03	1.15E-01	1.54E-02	2.21E-02
SRD						
Average	1.59E+01	7.75E-01	5.96E-01	9.63E+00	6.20E-01	8.97E-01
STD	3.10E+00	1.46E-01	1.56E-02	7.99E+00	5.23E-02	9.13E-02
WBM						
Average	1.41E-01	5.44E-01	6.77E-01	2.23E-01	6.84E-01	5.08E-01
STD	2.51E-02	6.63E-02	6.78E-03	1.34E-01	1.91E-03	4.35E-02

Source: Authors, (2026).

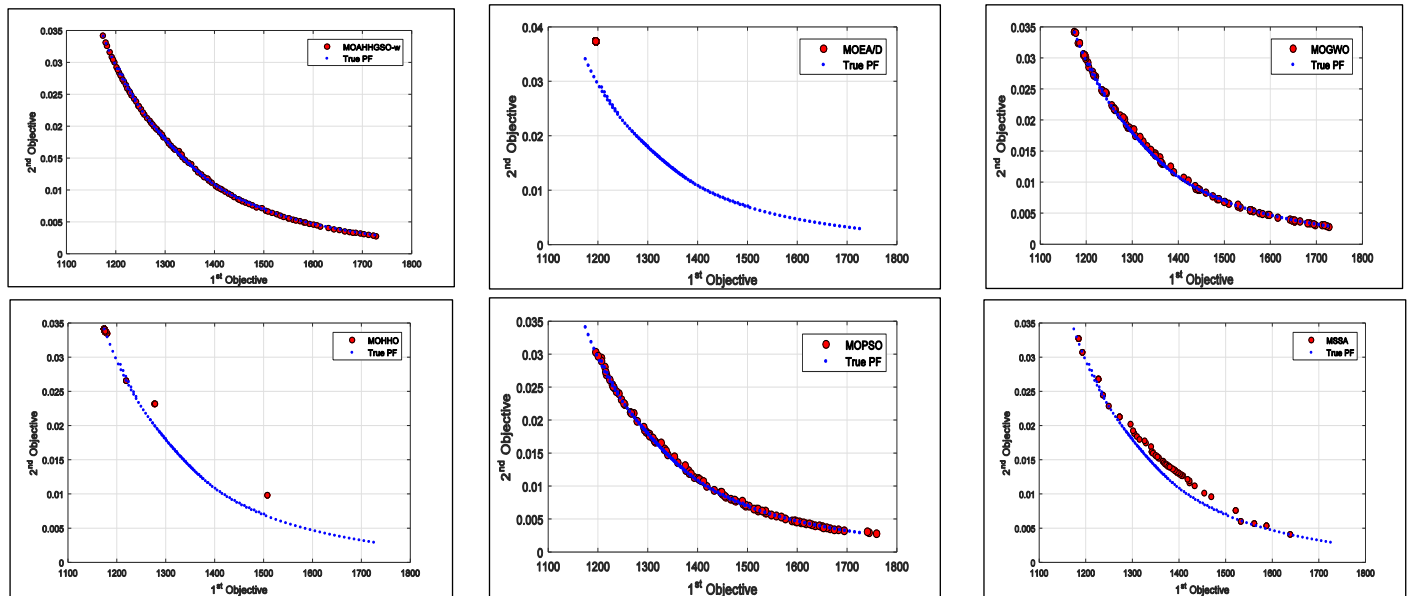


Figure 1: Obtained Pareto optimal solutions by MOAHHGSO-w, MOEA/D, MOGWO, MOHHO, MOPSO, and MSSA for the four-bar truss design problem.

Source: Authors, (2026).

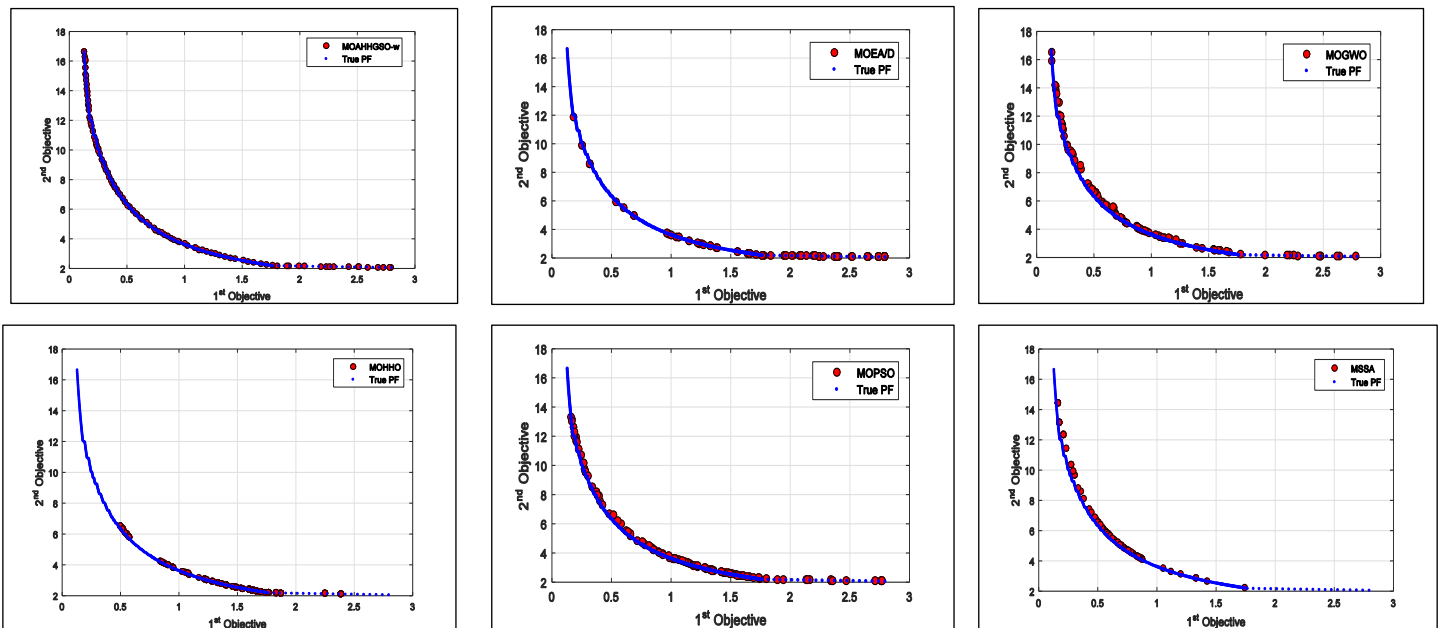


Figure 2: Obtained Pareto optimal solutions by MOAHHGSO-w, MOEA/D, MOGWO, MOHHO, MOPSO, and MSSA for the disk brake design problem.

Source: Authors, (2026).

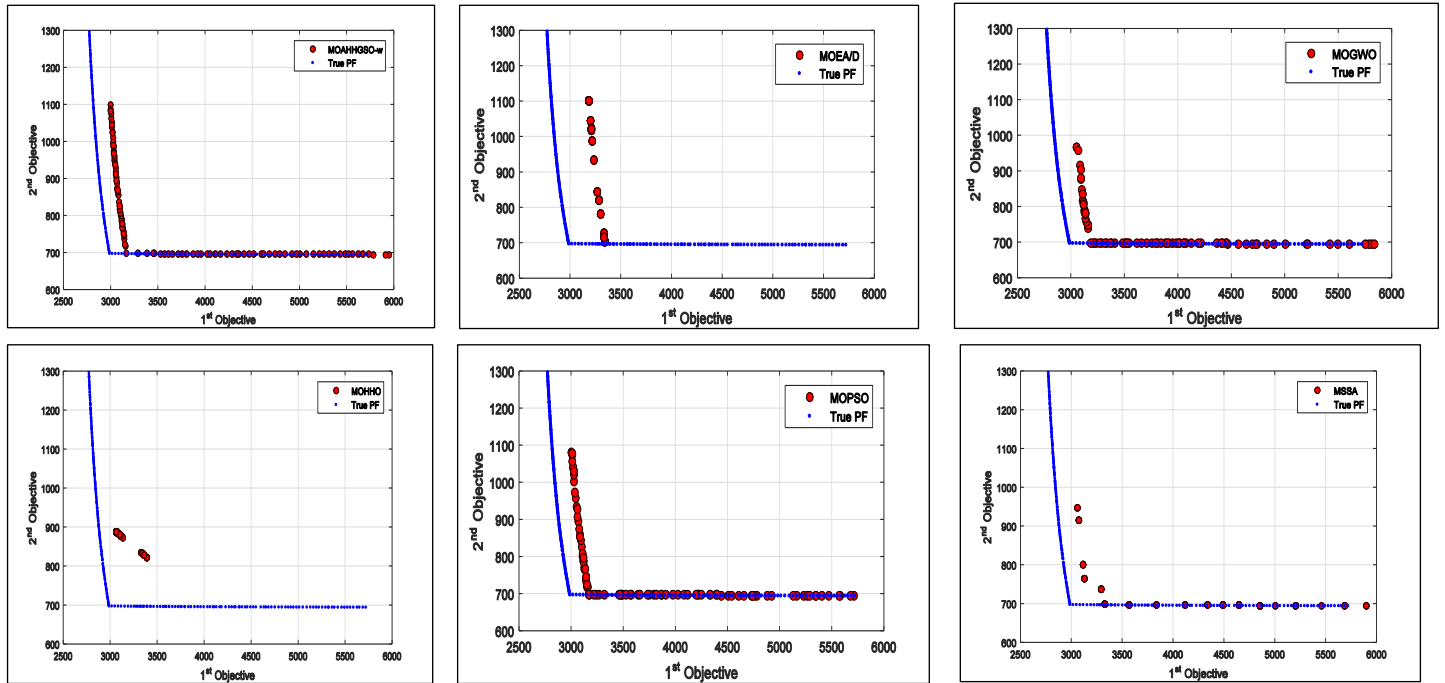


Figure 3: Obtained Pareto optimal solutions by MOAHHGSO-w, MOEA/D, MOGWO, MOHHO, MOPSO, and MSSA of the speed reducer design problem.
Source: Authors, (2026).

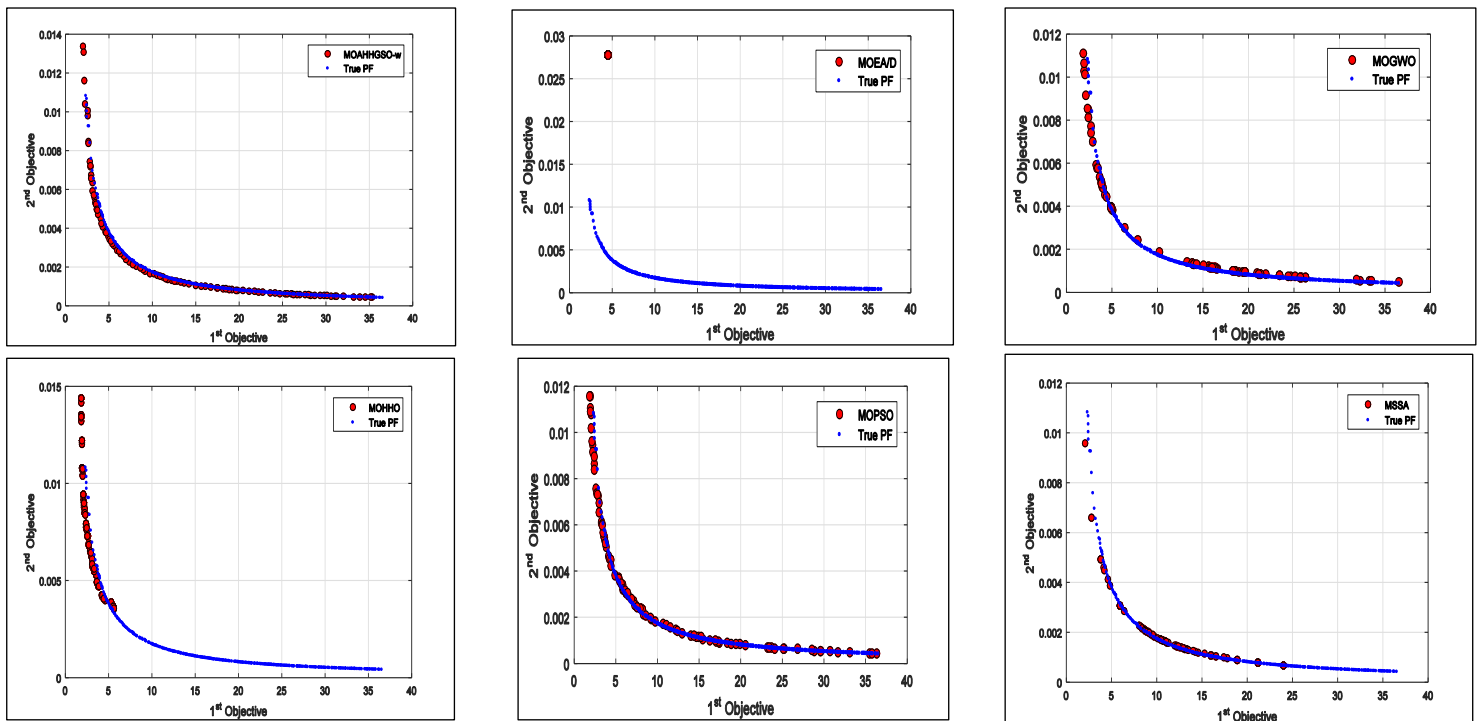


Figure 4: Obtained Pareto optimal solutions by MOAHHGSO-w, MOEA/D, MOGWO, MOHHO, MOPSO, and MSSA of the welded beam design problem.
Source: Authors, (2026).

IV.3 DISCUSSION OF THE RESULTS

The MOAHHGSO-w algorithm is a hybrid algorithm that combines two metaheuristic single-objective methods, the Artificial Hummingbird algorithm and the Henry Gas Solubility Optimization, with a multi-objective exact approach, the Weighted Sum algorithm. This proposed hybrid algorithm combines the characteristics and strengths of each algorithm integrated into it to provide better performance in the optimization process. In order to improve the suggested method for solving multi-objective optimization problems, the existing algorithms were improved and additional strategies were implemented, as previously described. The proposed MOAHHGSO-w algorithm is applied to different real engineering application. The proposed algorithm's superiority is evident both quantitatively and graphically.

It can be concluded that the MOAHHGSO-w algorithm effectively discovers a well-distributed set of non-dominated solutions with a strong convergence to the true Pareto front, even when dealing with two conflicting objectives in the search space. The findings suggest that the MOAHHGSO-w method effectively circumvents local optima Pareto fronts and successfully tackles high-dimensional issues with varied morphologies. The following comparison demonstrates that the MOAHHGSO-w algorithm is very competitive when compared to the other chosen algorithms. The statistical findings clearly demonstrate the exceptional performance of the MOAHHGSO-w algorithm. The factors contributing to this performance may be delineated as follows:

- The MOAHHGSO-w is attributed to the effective exploration techniques used by the AHA algorithm during the optimization process. This involves the hummingbird population alternating between the guided foraging strategy and the territorial foraging strategy; hence, it enhances the exploration of the search space. At the end of the optimization phase, hummingbirds use the migration foraging strategy to improve space search efficiency.
- The conventional HGSO algorithm has excellent performance. The strong updating mechanism, which relies on updated values of parameters $V_{i,j}$ and γ , demonstrates robust exploration and exploitation capabilities.
- The MOAHHGSO-w utilizes three types of external archive population to store the most optimal nondominated solutions achieved throughout the process optimization. This helps lead the MOAHHGSO-w in exploring the most productive areas of the search space.
- Utilizing ϵ -dominance to update the archive population enables the acquisition of a precise estimation of the Pareto set, prevents the archive from becoming too large, and decreases the execution time of the MOAHHGSO-w algorithm.
- Leaders selection strategy and selecting the target position strategy and using crowding distance in both of these strategies enables a favorable equilibrium between exploring and exploiting the search space.

Nevertheless, despite its efficiency, the suggested method incorporates several regulating factors, making it more susceptible and less adaptable to variations in these parameters. The crowding distance is a metric that prioritizes extreme points above solutions that provide a decent balance between goals during the optimization process. This issue may be resolved by using alternative diversity estimate techniques or by enhancing the used distance metric. Additional mechanisms and more efficient strategies may be included in the proposed algorithm to address multi-objective issues. This algorithm can be used for many optimization problems that need more investigation in future studies. To enhance the MOAHHGSO-w method for addressing the challenges associated with multi-objective optimization issues. It is important to use novel dominance relationships in order to differentiate among the non-dominated alternatives.

V. CONCLUSIONS

This research introduces a novel hybrid optimization approach, termed MOAHHGSO-w, which integrates features from the Artificial Hummingbird Algorithm (AHA), Henry Gas Solubility Optimization (HGSO), and the Weighted Sum method (WS) to effectively tackle complex multi-objective optimization problems. The proposed MOAHHGSO-w framework employs three distinct types of archives—local, global, and temporary—to efficiently handle multi-objective problems (MOPs). These outside archives are very important for keeping the balance between convergence and solution diversity. This step makes sure that the Pareto optimal front is very closely approximated while the solutions are spread out evenly. Moreover, the incorporation of the crowding distance mechanism enhances the diversity of non-dominated solutions during archive updates. Additionally, two key aspects of the proposed method include the strategy for determining the location of the target food source and the leader selection process. The method uses both epsilon dominance and crowding distance techniques to manage the population archives. These mechanisms prevent uncontrolled population growth and facilitate the generation of diverse solutions. The effectiveness of MOAHHGSO-w is assessed using four real-world engineering multi-objective design cases. The suggested hybrid method is also tested against five well-known multi-objective optimization methods. Experimental results indicate that the suggested method efficiently solves MOPs by demonstrating robust convergence and maintaining diversity in solutions. The findings point out the strengths of MOAHHGSO-w in addressing real-world, complex multi-objective optimization challenges. Future research will focus on extending this approach to handling many-objective optimization problems and investigating its practicality.

VI. AUTHOR'S CONTRIBUTION

Methodology: Amal Derreche and Djaafar Zouache.

Investigation: Amal Derreche and Djaafar Zouache.

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Resources: Amal Derreche.

Supervision: Amal Derreche and Boualem Brahmi.

Approval of the final text: Amal Derreche, Boualem Brahmi and Djaafar Zouache.

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