

ASSESSING DYNAMIC INSTABILITY USING HOPF BIFURCATION IN MULTI-MACHINE SYSTEMS EQUIPPED WITH POWER SYSTEM STABILIZER

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ABSTRACT

The small signal stability analysis is a pivotal tool for understanding low frequency oscillatory phenomenon and dynamic voltage instability. To address dynamic instability, new load flows were executed by increasing the load and generation profiles. These solutions are offered to linearization programs to investigate asymptotic stability of equilibrium points in nonlinear systems. Taking two cases and eight subcases in Kundur two area system, Hopf bifurcations were induced and corresponding bifurcation parameters were obtained with and without Power System Stabilizer (PSS) placement alongside static exciter. Capability of PSS in extending the load limits at various load buses was observed via Hopf bifurcation analysis. Performance of PSS in extending the dynamic stability limit at various load buses under single tie line contingency was also studied and it was found that PSS improved the dynamic power loading at those buses.



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I. INTRODUCTION

Low frequency oscillatory behaviour and dynamic instability phenomenon arising in power grids can be studied by using small signal analysis [1]. As power systems are inherently nonlinear, they are represented by Differential-Algebraic Equations (DAE's). It is necessary to use linear and nonlinear dynamical theories to study system behaviour. The concept of damping is central for understanding the stability of dynamical system. Low frequency oscillations pose threat to system behaviour. To improve the damping of low frequency oscillations, it has been suggested in [2], [3] to use Power System Stabilizer's (PSS) along with inherent Automatic Voltage Regulator (AVR). The design of PSS in power systems via comprehensive damping ratio maximization to simultaneously improve all the modal damping ratios are suggested in the works of [4]. Among them, local area modes whose frequency range is from 0.8 -2.5 Hz and the inter area modes whose frequency range is from 0.2-0.8 Hz needs to be studied for stabilizer design [3]. Although, there are many types of PSS architectures like PSS 2B, 3B, 4B [5], most of the references like [3], [6] have used two stage lead lag PSS topology for controller design. A brief review of PSS types and their tuning can be seen in [2]. These PSS models are combined with models of synchronous machines, AVR and network. Various types of generator models like 0.0, 1.0, 1.1, 2.1, 2.2, 3.2 and 3.3 are presented in literature. The number preceding the dot represents the number of windings on d-axis. The number succeeding the dot represents the number of windings on q-axis [7]. Multi machine systems with more load buses needs to be analysed because both small signal angle stability and voltage stability are to be examined. In this manuscript, the authors have chosen to use Kundur two area system shown in Figure 1 to study the above phenomenon. For linearization, Hartman–Grobman theorem [8] states that the stability of equilibrium points in a nonlinear system synthesised via linear analysis can be used to find the asymptotic stability of the same. If one of the poles of the linearized system cross the imaginary axis, into right half of S-plane while the other roots lie in the left half, such a phenomenon is known as Hopf bifurcation. The parameter which induces this phenomenon is known as bifurcation parameter. Applications of Hopf bifurcation are also observed in several engineering disciplines [9]. Hopf bifurcation-based DC-DC converter design for feeding Constant Power (CP) loads can be seen in [10]. Loads like constant horse power motors have inverse relation between speed and torque and hence their product i.e. output mechanical power becomes constant. If losses are neglected, the input power drawn becomes constant which reflects CP load behaviour in steady state.

Simulations of systems with multiple static and dynamic load models drawing power at load buses are found in [11]. Increasing available transfer capacity in Kundur two area system via Hopf and saddle node bifurcation details are studied by [12]. Authors of [12] considered 0.0 model for generator and constant impedance model for load for Hopf bifurcation analysis. [3] didn't consider the effects of Hopf bifurcation in Western States Coordinating Council System having renewable sources. They limited their discussions to prove the ability of PSS in minimizing time-domain metrics via artificial intelligence techniques. It was mentioned in [13] that [14] didn't consider the change in network topology for redesigning controls via Hopf bifurcation. Inspired from their reviews, this deficiency was also dwelled on authors of this manuscript by removing one tie line in service of Kundur two area system. In conventional power system stability program, most of the authors [15] treat loads as constant impedance as these values can be merged with the existing bus admittance matrix in a systematic manner. We have considered CP load model in this paper. Figure 1 shows typical diagram of the system having four generators (G_1, G_2, G_3, G_4) and two loads (A and B) connected through a transmission network. G_1, G_2 and G_3, G_4 are from area 1 and area 2 respectively. While [16] considered PSO-PSS at G_1 for damping ratio improvement via comprehensive index ratio maximization, the authors of this manuscript extended the study of Hopf bifurcations in the same system with PSO-PSS data. From the above literature, the following objectives are defined:

1. Improving the damping of inter area mode and other swing modes for congested and single tie line contingency cases by installing PSS besides AVR.
2. Exploring the dynamic instability phenomenon without PSS in Kundur two area system by treating loads as constant power instead of constant impedance as in [15].
3. Extending the dynamic instability through Hopf bifurcation analysis with PSS in the system for congested and single tie line contingency cases after installing PSS.

Industrial relevance of the above objectives is further detailed below:

1. In power industry, proper placement and sizing of PSS at the generating station improves the damping ratio of swing and interarea modes. Thus, if small oscillations build up in the inter area power flowing from one area to another area, these can be significantly damped using PSS.
2. Year after year, the utilities increase their generating capacities to match with the increase in the demanded power of the consumers. If the maximum loadability limits are exceeded there is a possibility for blackouts and this can be prevented by the power industry operators and planners.
3. If the maximum loadability limits can be extended by placing devices like PSS at the generating end, unnecessary load shedding, blackout prevention, transmission line and generator outages can be avoided to certain extent. This ensures continuous and reliable power supply which is central objective to power industry investors.

Case 1 and 2 deal with the congested and single tie line contingency cases. Here the variable ' λ ' stands for the loading parameter at the load bus 5 and 6. The following variations A-H is explored for assessing the dynamic instability phenomenon.

A, B: Variation in real power of load -A and load-B respectively without PSS.

C, D: Variation in λ corresponding to load-A and load-B respectively without PSS.

E, F: Variation in real power of load-A and load-B respectively with PSS.

G, H: Variation in λ corresponding to load-A and load-B respectively with PSS.

Section II deals with general details of system modelling and Hopf bifurcation. Section III deals with the results and discussions. Section IV provides conclusions of the study.

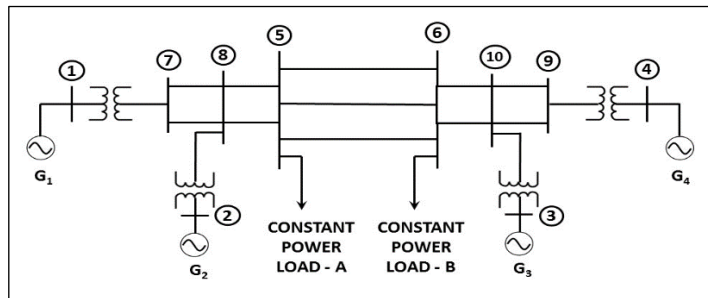


Figure 1: One line diagram of two area system.

Source: Authors, (2026).

II. SYSTEM MODELLING AND HOPF BIFURCATION

Section II.1 enumerates the modelling of Kundur two area system without any PSS. Section II.2 explains the modelling of power system stabilizer. Section II.3 details the general concepts of Hopf bifurcation and Section II.4 combines all the above via a detailed algorithm.

II.1 MODELLING OF KUNDUR TWO AREA SYSTEM

Details of modelling of Kundur two area system for linearization can be found in [1]. The nomenclature is taken from [1]. System has 10 nodes, 4 transformers and 11 lines. Buses 1-3 are PV buses. Bus no 4 is taken as slack bus. Nodes 5-10 are modelled as PQ buses. The 5th and 6th buses are having shunt capacitive susceptance of 3 and 4 p.u. All generators are modelled by two axis model i.e.1.1 whose details are found in [1].

Stator is represented by algebraic expressions instead of differential equations to simplify analysis. State variables of rotor circuits are taken as E'_q and E'_d instead of flux linkages to make analysis neat and elegant. Static exciter of single time constant and a large gain is modelled at each generator. Network is represented by medium π section quasi-steady state model in real and reactive power balance form. Loads are represented by CP absorbing type models in both dynamic and steady-state. Network transients are neglected as only low frequency modes are of concern. To study sub synchronous resonance phenomenon, it is vital to represent the network with the differential equations instead of steady state equations.

II.2 MODELLING OF POWER SYSTEM STABILIZER

Power system stabilizer (PSS) is an economic choice for damping improvement which is placed as an input to AVR block. It improves the damping as it is accommodated with lead lag blocks of time constants ' T_1 ', ' T_2 ', ' T_3 ' and ' T_4 '. To ensure that it only responds during transient conditions, a practical differentiator of gain ' K_s ' and time constant ' T_Q ' is connected in cascade to the above as shown in Figure 2. The PSS presented in this manuscript utilizes ' $\Delta\omega$ ' as control input. The value of washout time constant ' T_Q ' depends on the objective of compensation. If the objective is to improve the local mode of oscillation arising in single machine connected to infinite bus system, then ' T_Q ' is taken between 1-2 s [7]. For improving the damping ratio of inter area modes generated from multi-machine systems, value of the same needs to be chosen between 10-20 s [7].

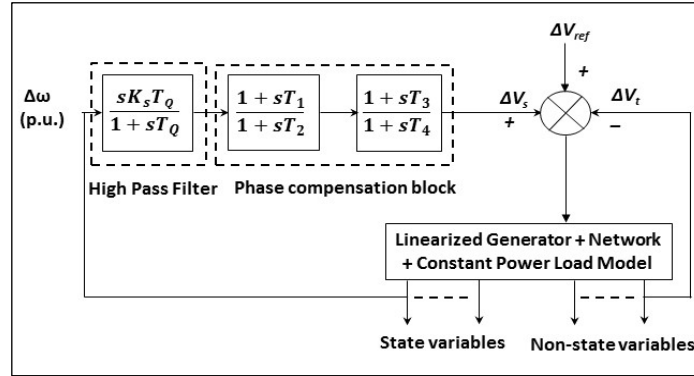


Figure 2: Block diagram of Power System Stabilizer (PSS).
Source: Authors, (2026).

II.3 HOPF BIFURCATION

Consider an autonomous system given by Equations (1-2). The order of vector ' x ' and ' y ' are ' k ' and ' r ' respectively. ' x ' is vector of state variables. Vector ' y ' represents algebraic variables. ' u ' is control variable and ' p ' is parametrized variable which brings bifurcation in the system.

$$\dot{x} = f(x, y, u) \quad (1)$$

$$g(x, y, p) = \in \dot{y} \quad (2)$$

For a given value of parameter ' p ', ' g ' can be solved to obtain the equilibrium point ' x_e '. Equations (1) and (2) are linearized about ' x_e ' to determine the asymptotic stability of the operating point. Equations (1-2) are linearized as follows for fixed ' u ' and initial operating point ' p^0 ' to get Equations (3-4).

$$\Delta \dot{x} = A_{f,x} \Delta x + A_{f,y} \Delta y \quad (3)$$

$$\in \Delta \dot{y} = A_{g,x} \Delta x + A_{g,y} \Delta y \quad (4)$$

Making $\in$ as zero and eliminating Δy , Equation (5) is obtained.

$$\Delta \dot{x} = \left[A_{f,x}^{p^0} - A_{f,y}^{p^0} A_{g,y}^{p^0 -1} A_{g,x}^{p^0} \right] \Delta x = \left[A_{sys}^{p^0} \right] \Delta x \quad (5)$$

Where each submatrix $A_{f,x}^{p^0}$, $A_{f,y}^{p^0}$, $A_{g,x}^{p^0}$, $A_{g,y}^{p^0}$ of Equation (5) are defined below.

$$A_{f,x}^{p^0} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_k} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_k}{\partial x_1} & \dots & \frac{\partial f_k}{\partial x_k} \end{bmatrix}_{p^0}$$

$$\begin{aligned}
 A_{f,y}^{p^0} &= \begin{bmatrix} \frac{\partial f_1}{\partial y_1} & \dots & \frac{\partial f_1}{\partial y_r} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_k}{\partial y_1} & \dots & \frac{\partial f_k}{\partial y_r} \end{bmatrix}_{p^0} \\
 A_{g,x}^{p^0} &= \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \dots & \frac{\partial g_1}{\partial x_k} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_k}{\partial x_1} & \dots & \frac{\partial g_k}{\partial x_k} \end{bmatrix}_{p^0} \\
 A_{g,y}^{p^0} &= \begin{bmatrix} \frac{\partial g_1}{\partial y_1} & \dots & \frac{\partial g_1}{\partial y_k} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_r}{\partial y_1} & \dots & \frac{\partial g_r}{\partial y_k} \end{bmatrix}_{p^0} \\
 A_{sys}^{p^0} &= \left[A_{f,x}^{p^0} - A_{f,y}^{p^0} A_{g,y}^{p^0 -1} A_{g,x}^{p^0} \right] \quad (6)
 \end{aligned}$$

Although the above partial derivatives in Equation (6) can be evaluated by using first and second order differencing approaches [1], the authors of this manuscript chose to evaluate the exact values of partial derivatives for the sake of accuracy and exactness to monitor the parameter at which Hopf bifurcation arrives.

II.4 STEPS TO ASSESS DYNAMIC INSTABILITY VIA HOPF BIFURCATION

1. Compute load flow using Newton Raphson method treating loads as CP absorbing type.
2. Use reference frame transformation to obtain initialized values of machine variables in ‘d-q’ frame as in [1].
3. Evaluate $A_{sys}^{p^0}$ by linearizing all the DAE’s of machine, AVR, network and loads clubbed together without any PSS. Find the eigenvalues of system matrix. Find the damping ratios of swing modes, inter area mode and other modes of specific interest.
4. Apply small change in the real power/real and reactive power demanded by load with rescheduled generation and evaluate steps (1)-(3). The changes that needs to be done in load flow programme to accomplish this is detailed in [14].

$$P_{Li} = P_{Li,o}(\lambda_i) \quad (7)$$

$$Q_{Li} = Q_{Li,o}(\lambda_i) \quad (8)$$

Where,

λ_i is a loading factor introduced to bring Hopf bifurcation at i^{th} node.

P_{Li} , Q_{Li} are real and reactive power drawn by i^{th} load bus at each equilibrium point and

$P_{Li,o}$, $Q_{Li,o}$ are nominal real and reactive powers consumed by loads ‘A’ and ‘B’ residing at nodes 5 and 6 respectively.

5. Identify the critical mode which is crossing the right half of complex plane. Stop the algorithm and note down the loading at which dynamic instability is occurring.
6. Place PSS at ‘G₁’ [16], evaluate steps (1-5) and compare the ability of PSS in extending the dynamic instability through Hopf bifurcation.
7. Repeat steps (1-6) for single tie line network contingency and sketch the loci of inter area mode.

III. RESULTS AND DISCUSSIONS

The following programs were developed in MATLAB 2020 environment for achieving objectives defined in the section I.

1. Newton Raphson programme to find power flows and steady state network voltages and currents for congested tie line case. The power residuals were taken as 10^{-12} .
2. Eigenvalue analysis programme with AVR only.
3. Eigenvalue analysis programme with PSS at ‘G₁’.
4. Programme to identify critical mode upon application of variations defined by Cases A-H defined in section I.
5. Modification of programmes (1-4) for single tie line contingency case.

III.1 DISCUSSION REGARDING CASE 1

The line, load, machine and exciter data from [7] is taken in this study. The load flow solution presented in [7] is obtained exactly by the authors of this manuscript. In the machine data of [7], the mechanical damping of all machines is made as 0.01 to see that no zero eigenvalue occurs. PSS parameters at ‘G₁’ for Case 1 are taken as $T_1 = 0.2473$ s, $T_2 = 0.0218$ s, $T_3 = 3$ s, $T_4 = 5.4$ s, $T_Q = 10$ s, $K_s = 17.2079$ [16]. From Table 1, it is observed that placement of PSS at ‘G₁’ increased the damping ratio of interarea mode from 0.96% to 5.72%.

The damping ratio of swing mode 'a' and 'b' increased from 10.49% to 33.16% and 10.74% to 11.03% respectively thus fulfilling the 3% minimum damping ratio criteria suggested in [17]. To explore the concepts of Hopf bifurcation, subcases '1-A' to '1-H' are generated and presented in Table 2 showing the respective loadings at which dynamic instability is achieved. For Case 1-A, the instability occurs at a loading of 13.49 p.u.. With the placement of PSS, the dynamic instability occurs at 19.02 p.u. thus proving the extension of maximum loadability. For Case 1-E, it is not the inter area mode which is unstable, but it is the high frequency mode 'd' shown in bold in Case 1-E of Table 2. The dynamic instability is happening at a loading of 17.3 p.u. for Case 1-B but after PSS placement, the loading is extended till 21.06 p.u.. The mode which is undergoing instability is the mode 'd'.

Table 1: Eigenvalues at nominal loading with and without PSS for Cases 1, 2 respectively.

Eigenvalues			
Case 1		Case 2	
Without PSS	With PSS at G_1	Without PSS	With PSS at G_1
-0.0441 ± 4.6056i^a	-0.2615 ± 4.5606i^a	-0.0332 ± 4.0895i^a	-0.1694 ± 4.0517i^a
-0.7680 ± 7.2818i ^b	-2.2872 ± 6.5073i ^b	-0.7622 ± 7.2502i ^b	-2.2518 ± 6.7571i ^b
-0.7179 ± 6.6473i ^c	-0.7385 ± 6.6537i ^c	-0.7064 ± 6.6253i ^c	-0.7154 ± 6.6263i ^c
-23.4941 ± 43.2130i	-23.3135 ± 43.2623i	-23.5362 ± 43.5700i	-23.3927 ± 43.6103i
-24.9842 ± 15.2554i	-21.1613 ± 17.7450i	-24.8304 ± 19.1723i	-22.8324 ± 20.2304i

a) Interarea mode, b) & c) – Swing mode of area 1 and 2.

Source: Authors, (2026).

To explore the concepts of Hopf bifurcation, subcases 1-A to 1-H are generated and results are presented in Table 2. For Case 1-A, the dynamic instability occurs at a loading of 13.49 p.u. With the placement of PSS, the dynamic instability occurs at 19.02 p.u., thus extending the maximum loading limit at 5th node. The percentage improvement in loadability limit with PSS in this subcase is 41%. For Case 1-E, it is not the inter area mode which is unstable, but it is the high frequency mode 'd' shown in bold in Case 1-E of Table 2. The dynamic instability is happening at a loading of 17.3 p.u. for Case 1-B, but after PSS placement, the loading is extended till 21.06 p.u.. The mode 'd' is undergoing instability in this subcase. The percentage improvement in loadability limit with PSS in this subcase is 21.73%.

Table 2: Eigenvalues with and without PSS for Case 1 near critical loading.

Eigenvalues without PSS			
Case 1-A, $P_5=13.49$ p.u.	Case 1-B, $P_6=17.30$ p.u.	Case 1-C, $\lambda_5=1.1755$	Case 1-D, $\lambda_6=1.1015$
-0.0002 ± 4.6742i^a	-0.0001 ± 4.5898i^a	-0.0004 ± 4.6586i^a	-0.0002 ± 4.5747i^a
-0.6788 ± 7.3416i ^b	-0.6954 ± 7.3350i ^b	-0.6709 ± 7.3331i ^b	-0.6925 ± 7.3333i ^b
-0.6121 ± 6.7448i ^c	-0.6152 ± 6.7090i ^c	-0.6031 ± 6.7488i ^c	-0.6082 ± 6.7049i ^c
-23.0260 ± 48.4425i	-22.9913 ± 49.7915i	-22.9420 ± 49.8976i	-22.9076 ± 51.2536i
-24.9684 ± 15.4713i	-24.9777 ± 15.6667i	-24.9655 ± 15.7650i	-24.9774 ± 15.8534i
Eigenvalues with PSS at G_1			
Case 1-E, $P_5=19.02$ p.u.	Case 1-F, $P_6=21.06$ p.u.	Case 1-G, $\lambda_5=1.5715$	Case 1-H, $\lambda_6=1.2965$
-0.0057 ± 4.5918i ^a	-0.1426 ± 4.3611i ^a	-0.0520 ± 4.5681i ^a	-0.1541 ± 4.3551i ^a
-2.0015 ± 6.6299i ^b	-1.9756 ± 6.7009i ^b	-1.9398 ± 6.6080i ^b	-2.0001 ± 6.6740i ^b
-0.3390 ± 6.9652i ^c	-0.3700 ± 6.8395i ^c	-0.3735 ± 6.9336i ^c	-0.4006 ± 6.8142i ^c
-21.9514 ± 19.2332i	-21.3567 ± 19.4759i	-22.1115 ± 19.4797i	-21.4284 ± 19.5598i
-20.811 ± 6.8686i	-19.7560 ± 6.7009i	-20.8395 ± 7.3090i	-20.0016 ± 6.6740i
0.3656 ± 174.0255i^d	0.5750 ± 178.3604i^d	0.2895 ± 176.8891i^d	0.3335 ± 180.1069i^d

a) – Interarea mode, b) – Swing mode of area 1, c) – Swing mode of area 2, d) – High frequency mode.

Source: Authors, (2026).

Table 3: Eigenvalues with and without PSS for Case 2 near critical loading.

Eigenvalues without PSS			
Case 2-A, $P_5=14.07$ p.u.	Case 2-B, $P_6=17.19$ p.u.	Case 2-C, $\lambda_5=1.2275$	Case 2-D, $\lambda_6=1.0915$
-0.0002 ± 4.1967i^a	-0.0003 ± 4.0313i^a	-0.0006 ± 4.1763i^a	-0.0003 ± 4.0178i^a
-0.6455 ± 7.3289i ^b	-0.6945 ± 7.2988i ^b	-0.6350 ± 7.3191i ^b	-0.6933 ± 7.2973i ^b
-0.5728 ± 6.7512i ^c	-0.6069 ± 6.6846i ^c	-0.5622 ± 6.7567i ^c	-0.6019 ± 6.6804i ^c
-22.8851 ± 50.5957i	-23.0476 ± 50.3710i	-22.7558 ± 52.6631i	-22.9764 ± 51.6880i
-24.7706 ± 19.8780i	-24.8092 ± 19.8388i	-24.7644 ± 20.3346i	-24.8094 ± 19.9920i
Eigenvalues with PSS at G_1			
Case 2-E, $P_5=18.41$ p.u.	Case 2-F, $P_6=20.43$ p.u.	Case 2-G, $\lambda_5=1.5690$	Case 2-H, $\lambda_6=1.2620$
0.0005 ± 4.0660i^a	-0.0650 ± 3.6657i ^a	-0.0111 ± 4.0176i ^a	-0.0757 ± 3.6927i ^a
-1.9902 ± 6.8743i ^b	-1.9678 ± 6.9041i ^b	-1.9017 ± 6.8440i ^b	-1.9914 ± 6.8831i ^b
-0.3462 ± 6.9204i ^c	-0.3701 ± 6.8089i ^c	-0.3494 ± 6.9111i ^c	-0.3969 ± 6.7859i ^c
-23.3188 ± 23.7304i	-22.8626 ± 23.0627i	-23.4539 ± 24.5583i	-22.8881 ± 23.0217i
-19.9085 ± 6.0249i	-1.9678 ± 6.9041i	-19.9528 ± 6.7644i	-20.6529 ± 3.6242i
-15.3129 ± 109.0803i ^d	0.4901 ± 183.9575i^d	0.4709 ± 179.9221i^d	2.9495 ± 194.7850i^d

a) – Interarea mode, b) – Swing mode of area 1, c) – Swing mode of area 2, d) – High frequency mode.

Source: Authors, (2026).

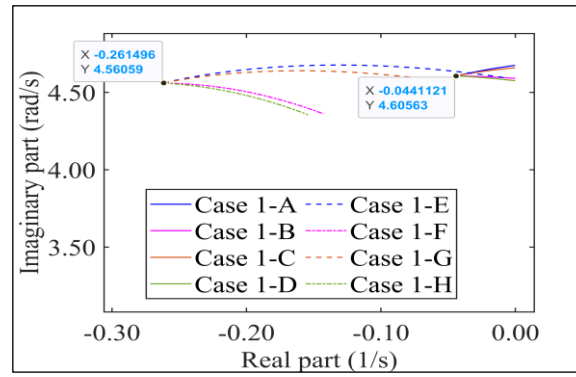


Figure 3: Loci of interarea mode corresponding to Case 1.
Source: Authors, (2026).

For Case 1-C, inter area mode is the critical mode occurring at λ_5 value of 1.1755. After placing PSS, the dynamic instability is extended till a value of 1.5715. Here, it is the mode 'd' which is undergoing Hopf bifurcation instead of mode 'a'. The percentage improvement in loadability limit with PSS in this subcase is 33.69%. The critical loading λ_6 is 1.1015 for Case 1-D. In this sub-case, the critical mode is mode 'a'. After installing PSS (Case 1-H), the critical loading is put off till 1.2965 and even in this case it is the mode 'd' undergoing Hopf Bifurcation. The percentage improvement in loadability limit with PSS in this subcase is 17.70%. The loci of inter-area mode 'a' with and without placing PSS after applying all subcases till critical loading is depicted in Figure 3 for Case 1.

III.2 DISCUSSION REGARDING CASE 2

The values of PSS parameters at G_1 for Case 2 were taken as $T_1 = 0.2173$ s, $T_2 = 0.0218$ s, $T_3 = 3$ s, $T_4 = 5.4$ s, $T_Q = 10$ s, $K_s = 16.6370$ [16]. From Table 1, it is observed that with placement of PSS at G_1 , there is an improvement of damping ratio of mode 'a' from 0.81% to 4.18%. The damping ratio increased from 10.46% to 31.62%, 10.60% to 10.73% for swing mode 'a' and 'b' respectively. There is significant improvement in damping ratio of mode 'a' and 'b' but not 'c'. But all modes have damping ratio values greater than 3% as suggested by [16], [17]. To explore dynamic instability, subcases from 2-A to H are generated and presented in Table 3 showing the respective loadings at which dynamic instability is achieved. For Case 2-A, the dynamic instability occurs at a loading of 14.07 p.u. in the form of mode 'a'. With the placement of PSS, the dynamic instability occurs at 18.41 p.u., thus ascertaining the extension of load limit. The percentage improvement in loadability limit with PSS in this subcase is 30.85%. The dynamic instability is happening at a loading of 17.19 p.u. for Case 2-B but after PSS placement, the loading can be extended till 20.43 p.u. The mode which is undergoing instability is the mode 'd'. The percentage improvement in loadability limit with PSS in this subcase is 18.85%. For Case 2-C, mode 'a' is the critical mode occurring at λ_5 value of 1.2275. After placing PSS, the dynamic instability can be extended till a value of 1.5690. For this case, it is the mode 'd' which is undergoing Hopf bifurcation instead of mode 'a'. The percentage improvement in loadability limit with PSS in this subcase is 27.82%. The critical loading λ_6 is 1.0915 for Case 2-D. The critical mode is mode 'a'. After installing PSS (Case 2-H), the critical loading λ_6 is put off till 1.2620 and in this case, it is the mode 'd' which is the critical mode. The percentage improvement in loadability limit with PSS in this subcase is 15.62%. The loci of inter-area mode 'a' with and without placing PSS after applying all subcases till critical loading is depicted in Figure 4 for Case 2.

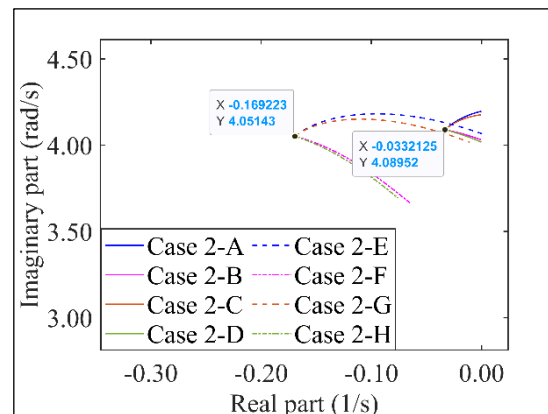


Figure 4: Loci of interarea mode corresponding to Case 2.
Source: Authors, (2026).

IV. CONCLUSIONS

A nonlinear autonomous system can be linearized about an equilibrium point for studying whether the system is asymptotically stable or not. Multiple equilibrium points are generated by changing the values of loads and generation of Kundur two area system. For each generated equilibrium point, the procedure of linearization is done to see whether at least one of the complex conjugate pair crosses the imaginary axis from left half of complex plane.

In order to have detailed view, two cases and eight subcases are sprouted to draw the loci of interarea mode of system. It was obtained that with the placement of PSS, dynamic instability can be extended to a higher loading as compared to the one obtained without PSS for both congested and single tie line contingency cases.

V. AUTHOR'S CONTRIBUTION

Conceptualization: Ghousul Azam Shaik and Lakshmi Devi Aithepalli.

Methodology: Ghousul Azam Shaik and Lakshmi Devi Aithepalli.

Investigation: Ghousul Azam Shaik and Lakshmi Devi Aithepalli.

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